PHILOSOPHICAL TRANSACTIONS.

I.—*Researches in Physical Astronomy.* *By* J. W. Lubbock, *Esq. V.P. and Treas. R.S.*

Read November 17, 1831.

On the Theory of the Moon.

In the following paper I have given the developments which are required in the Theory of the Moon when the square of the disturbing function is retained. These expressions result from the multiplication of series, each consisting of many terms; but they are formed with great facility by means of the second Table given in my former paper on the Lunar Theory.

I have not attempted the numerical calculation of the coefficients of the inequalities according to the method here explained, at least in the second approximation; but this work, which would tend to perfect the Tables of the Moon, is a desideratum in physical astronomy. The calculations will not I think be found longer than in the method of Clairaut, nor than those which are required in several astronomical problems. The developments which I have given ought however to be verified in the first instance, although I have taken great pains to ensure their accuracy.

With respect to the convergence of the expressions, it may be remarked that when the same powers of the eccentricities are retained, the results must be identical, whichever method be employed. If part of the coefficients of the terms already considered due to the higher powers of the eccentricities are sensible, it follows that other arguments must be considered in addition to those introduced by M. Damoiseau; and conversely if the arguments which

M. DAMOISEAU has considered are sufficient, it is unnecessary in either method to carry the approximation beyond the fourth power of the eccentricity of the Moon, and quantities of that order.

The method I have employed is equally advantageous in the first approximation. I have given in conclusion the numerical results which are obtained of the coefficients of the principal inequalities when the square of the disturbing function is not considered, which may be regarded as an elementary Theory of the Moon; for the differential equations and the equations which serve to determine the coefficients retain nearly the same form in the further approximations.

The coefficient of the *variation* obtained in this manner differs only by a few seconds from that given by NEWTON in the third volume of the Principia; that of the *evection* agrees closely with the value assigned to it by M. DAMOISEAU. This latter agreement of course can only be looked upon as accidental.

Developments required for the integration of the equation

$$\frac{\mathrm{d}^2 \mathrm{r}^2}{2\,\mathrm{d}\,t^2} - \frac{\mathrm{d}^2\,\mathrm{r}^3\,\delta\,\frac{1}{r}}{\mathrm{d}\,t^2} + \frac{3\,\mathrm{d}^2\,.\,\mathrm{r}^4\left(\delta\,\frac{1}{r}\right)^2}{2\,\mathrm{d}\,t^2} - \frac{\mu}{r} + \frac{\mu}{a} + 2\int \mathrm{d}\,R + r\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right) = 0$$

when the square of the disturbing force is retained.

$$\text{Since } \mathrm{r} = \underset{[0]}{1 + \frac{e^2}{2}} - \underset{[2]}{e\left(1 - \frac{3}{8}\,e^2\right)\cos x} - \underset{[8]}{\frac{e^2}{2}\left(1 - \frac{2\,e^2}{3}\right)\cos 2\,x} - \underset{[20]}{\frac{3\,e^3}{8}\cos 3\,x} - \underset{[38]}{\frac{e^4}{3}\cos 4\,x}$$

$$\mathrm{r}\,\delta\,\frac{1}{r} = \left\{\left(1 + \frac{e^2}{2}\right) r_1 - \frac{e^2}{2}\left(1 - \frac{3}{8}\,e^2\right)\left\{r_3 + r_4\right\} - \frac{e^4}{4}\left\{r_9 + r_{10}\right\}\right\}\underset{[1]}{\cos 2\,t}$$

$$+ \left\{\left(1 + \frac{e^2}{2}\right) r_2 - \frac{1}{2}\left(1 - \frac{3}{8}\,e^2\right)\left\{2\,r_0 + e^2\,r_8\right\} - \frac{e^2}{4}\,r_2\right\}\underset{[2]}{e\cos x}$$

$$+ \left\{\left(1 + \frac{e^2}{2}\right) r_3 - \frac{1}{2}\left(1 - \frac{3}{8}\,e^2\right)\left\{e^2\,r_9 + r_1\right\} - \frac{e^2}{4}\,r_4\right\}\underset{[3]}{e\cos(2\,t - x)}$$

$$+ \left\{\left(1 + \frac{e^2}{2}\right) r_4 - \frac{1}{2}\left(1 - \frac{3}{8}\,e^2\right)\left\{r_1 + e^2\,r_{10}\right\} - \frac{e^2}{4}\,r_3\right\}\underset{[4]}{e\cos(2\,t + x)}$$

$$+ \left\{\left(1 + \frac{e^2}{2}\right) r_5 - \frac{1}{2}\left(1 - \frac{3}{8}\,e^2\right)\left\{e^2\,r_{14} + e^2\,r_{11}\right\}\right\}\underset{[5]}{e_{\prime}\cos z}$$

$$+\left\{\left(1+\frac{e^2}{2}\right)r_6-\frac{1}{2}\left(1-\frac{3}{8}e^2\right)\left\{e^2 r_{12}+e^2 r_{16}\right\}\right\}e_{,}\cos(2t-z)$$
[6]

$$+\left\{\left(1+\frac{e^2}{2}\right)r_7-\frac{1}{2}\left(1-\frac{3}{8}e^2\right)\left\{e^2 r_{15}+e^2 r_{13}\right\}\right\}e_{,}\cos(2t+z)$$
[7]

$$+\left\{\left(1+\frac{e^2}{2}\right)r_8-\frac{1}{2}\left(1-\frac{3}{8}e^2\right)\left\{r_2+e^2 r_{20}\right\}-\frac{3}{16}e^2 r_2\right\}e^2\cos 2x$$
[8]

$$+\left\{\left(1+\frac{e^2}{2}\right)r_9-\frac{1}{2}\left(1-\frac{3}{8}e^2\right)\left\{e^2 r_{21}+r_3\right\}-\frac{r_1}{4}-\frac{3}{16}e^2 r_4\right\}e^2\cos(2t-2x)$$
[9]

$$+\left\{\left(1+\frac{e^2}{2}\right)r_{10}-\frac{1}{2}\left(1-\frac{3}{8}e^2\right)\left\{r_4+e^2 r_{22}\right\}-\frac{r_1}{4}-\frac{3}{16}e^2 r_3\right\}e^2\cos(2t+2x)$$
[10]

$$+\left\{\left(1+\frac{e^2}{2}\right)r_{11}-\frac{1}{2}\left(1-\frac{3}{8}e^2\right)\left\{r_5+e^2 r_{23}\right\}-\frac{e^2}{4}r_{14}\right\}e\,e_{,}\cos(x+z)$$
[11]

$$+\left\{\left(1+\frac{e^2}{2}\right)r_{12}-\frac{1}{2}\left(1-\frac{3}{8}e^2\right)\left\{e^2 r_{24}+r_6\right\}-\frac{e^2}{4}r_{16}\right\}e\,e_{,}\cos(2t-x-z)$$
[12]

$$+\left\{\left(1+\frac{e^2}{2}\right)r_{13}-\frac{1}{2}\left(1-\frac{3}{8}e^2\right)\left\{r_7+e^2 r_{25}\right\}-\frac{e^2}{4}r_{15}\right\}e\,e_{,}\cos(2t+x+z)$$
[13]

$$+\left\{\left(1+\frac{e^2}{2}\right)r_{14}-\frac{1}{2}\left(1-\frac{3}{8}e^2\right)\left\{e^2 r_{26}+r_5\right\}\right\}e\,e_{,}\cos(x-z)$$
[14]

$$+\left\{\left(1+\frac{e^2}{2}\right)r_{15}-\frac{1}{2}\left(1-\frac{3}{8}e^2\right)\left\{e^2 r_{27}+r_7\right\}\right\}e\,e_{,}\cos(2t-x+z)$$
[15]

$$+\left\{\left(1+\frac{e^2}{2}\right)r_{16}-\frac{1}{2}\left\{1-\frac{3}{8}e^2\right\}\left\{r_6+e^2 r_{28}\right\}\right\}e\,e_{,}\cos(2t+x-z)$$
[16]

$$+\left\{\left(1+\frac{e^2}{2}\right)r_{17}-\frac{1}{2}\left\{1-\frac{3}{8}e^2\right\}\left\{e^2 r_{32}+e^2 r_{29}\right\}\right\}e_{,}^2\cos 2z$$
[17]

$$+\left\{\left(1+\frac{e^2}{2}\right)r_{18}-\frac{1}{2}\left\{1-\frac{3}{8}e^2\right\}\left\{e^2 r_{30}+e^2 r_{34}\right\}\right\}e_{,}^2\cos(2t-2z)$$
[18]

$$+\left\{\left(1+\frac{e^2}{2}\right)r_{19}-\frac{1}{2}\left\{1-\frac{3}{8}e^2\right\}\left\{e^2 r_{33}+e^2 r_{31}\right\}\right\}e_{,}^2\cos(2t+2z)$$
[19]

+ &c. &c.

From the preceding development, that of $r^3 \delta . \frac{1}{r}$ may be immediately inferred.

$$\mathrm{r}^3 = 1 + 3e^2\left(1+\frac{e^2}{8}\right) - 3e\left(1+\frac{3}{8}e^2\right)\cos x - \frac{5}{8}e^4\cos 2x + \frac{e^3}{8}\cos 3x + \frac{e^4}{8}\cos 4x$$
$$[0] \qquad [2] \qquad [8] \qquad [20] \qquad [38]$$

The following approximate value of $\mathrm{r}\,\delta\frac{1}{r}$ will probably be found sufficient.

$$\mathrm{r}\,\delta\frac{1}{r} = \left\{\left(1+\frac{e^2}{2}\right)r_1 - \frac{e^2}{2}\left(r_3+r_4\right)\right\}\cos 2t + \left\{r_2 - r_0\right\}e\cos x$$
$$[1] \qquad [2]$$

$$+ \left\{r_3 - \frac{r_1}{2}\right\}e\cos(2t-x) + \left\{r_4 - \frac{r_1}{2}\right\}e\cos(2t+x)$$
$$[3] \qquad [4]$$

$$+ r_5 e_{\prime}\cos z + r_6 e_{\prime}\cos(2t-z) + r_7 e_{\prime}\cos(2t+z) + \left\{r_8 - \frac{r_1}{2}\right\}e^2\cos 2x$$
$$[5] \qquad [6] \qquad [7] \qquad [8]$$

$$+ \left\{r_9 - \frac{r_3}{2} - \frac{r_1}{4}\right\}e^2\cos(2t-2x) + \left\{r_{10} - \frac{r_4}{2} - \frac{r_1}{4}\right\}e^2\cos(2t+2x)$$
$$[9] \qquad [10]$$

$$+ \left\{r_{11} - \frac{r_5}{2}\right\}e\,e_{\prime}\cos(x+z) + \left\{r_{12} - \frac{r_6}{2}\right\}e\,e_{\prime}\cos(2t-x-z)$$
$$[11] \qquad [12]$$

$$+ \left\{r_{13} - \frac{r_7}{2}\right\}e\,e_{\prime}\cos(2t+x+z) + \left\{r_{14} - \frac{r_5}{2}\right\}e\,e_{\prime}\cos(x-z)$$
$$[13] \qquad [14]$$

$$+ \left\{r_{15} - \frac{r_7}{2}\right\}e\,e_{\prime}\cos(2t-x+z) + \left\{r_{16} - \frac{r_6}{2}\right\}e\,e_{\prime}\cos(2t+x-z)$$
$$[15] \qquad [16]$$

$$+ r_{17}e_{\prime}^2\cos 2z + r_{18}e_{\prime}^2\cos(2t-2z) + r_{19}e_{\prime}^2\cos(2t+2z)$$
$$[17] \qquad [18] \qquad [19]$$

$$a^2\left(\delta\frac{1}{r}\right)^2 = r_0^2 + \frac{r_1^2}{2} + \frac{e^2 r_2^2}{2} + \frac{e^2 r_3^2}{2} + \frac{e^2 r_4^2}{2} + \frac{e_{\prime}^2 r_5^2}{2} + \frac{e_{\prime}^2 r_6^2}{2} + \frac{e_{\prime}^2 r_7^2}{2}$$
$$[0]$$

$$+ \{2r_0 r_1 + e^2(r_3+r_4)r_2 + e_{\prime}^2(r_6+r_7)r_5\}\cos 2t + \{(r_4+r_3)r_1 + 2r_0 r_2\}e\cos x$$
$$[1] \qquad [2]$$

$$+ \{r_1 r_2 + 2r_0 r_3\}e\cos(2t-x) + \{r_1 r_2 + 2r_0 r_4\}\cos(2t+x)$$
$$[3] \qquad [4]$$

$$+ \{r_1 r_7 + r_1 r_6 + 2r_0 r_5\}e_{\prime}\cos z + \{r_5 r_1 + 2r_0 r_6\}e_{\prime}\cos(2t-z)$$
$$[5] \qquad [6]$$

$$+ \{r_5 r_1 + 2 r_0 r_7\} e_, \cos (2t + z) + \{r_2^2 + r_4 r_3 + r_1 r_9 + r_1 r_{10}\} e^2 \cos 2x$$
[7] [8]

$$+ \{r_2 r_3 + 2 r_0 r_9\} e^2 \cos (2t - 2x) + \{r_4 r_2 + 2 r_0 r_{10}\} e^2 \cos (2t + 2x)$$
[9] [10]

$$+ \{r_1 r_{13} + r_1 r_{12} + r_2 r_5 + r_6 r_4 + r_3 r_7 + 2 r_0 r_{11}\} e e_, \cos (x + z)$$
[11]

$$+ \{r_{11} r_1 + r_2 r_6 + r_5 r_3 + 2 r_0 r_{12}\} e e_, \cos (2t - x - z)$$
[12]

$$+ \{r_{11} r_1 + r_2 r_7 + r_5 r_4 + 2 r_0 r_{13}\} e e_, \cos (2t + x + z)$$
[13]

$$+ \{r_{16} r_1 + r_{15} r_1 + r_2 r_5 + r_6 r_3 + r_7 r_4 + 2 r_0 r_{14}\} e e_, \cos (x - z)$$
[14]

$$+ \{r_{14} r_1 + r_2 r_7 + r_5 r_3\} e e_, \cos (2t - x + z) + \{r_{14} r_1 + r_2 r_6 + r_5 r_4\} e e_, \cos (2t + x - z)$$
[15] [16]

$$+ \{r_5^2 + r_7 r_6 + r_1 r_{18} + r_1 r_{19}\} e_,^2 \cos 2z + \{r_{17} r_1 + r_5 r_6\} e_,^2 \cos (2t - 2z)$$
[17] [18]

$$+ \{r_{17} r_1 + r_7 r_5\} e_,^2 \cos (2t + 2z) + \frac{r_1^2}{2} \cos 4t + \frac{r_3^2}{2} \cos (4t - 2x)$$
[19] [131] [132]

From the preceding development that of $r^4 \left(\delta \frac{1}{r}\right)^2$ may be easily inferred.

$$r^4 = a^4 \{1 + 5e^2 - 4e \cos x + e^2 \cos 2x\}$$
[0] [2] [8]

Considering the terms only in R multiplied by $\frac{a^2}{a_,^3}$

$$R = - m_, \left\{ \frac{r'^2}{4 r_,^3} \{1 + 3 \cos (2\lambda - 2\lambda_,) - 2s^2\} \right\}$$

$$= - m_, \left\{ \frac{r^2}{4(1 + s^2) r_,^3} \{1 + 3 \cos (2\lambda - 2\lambda_,) - 2s^2\} \right\}$$

neglecting s^4

$$= - m_, \left\{ \frac{r^2}{4 r_,^3} \{1 + 3 \cos (2\lambda - 2\lambda_,)\} \{1 - s^2\} - \frac{r^2}{2 r_,^3} s^2 \right\}$$

$$\frac{dR}{ds} = m_, \left\{ \frac{r^2}{r_,^3} + \frac{r^2}{2 r_,^3} \{1 + 3 \cos (2\lambda - 2\lambda_,)\} \right\} s$$

$$\frac{r^2}{2\,r_{\prime}^3}{}^{*} + \frac{r^2}{4\,r_{\prime}^3}\{1 + 3\cos(2\lambda - 2\lambda_{\prime})\}$$

$$= \frac{a^2}{a_{\prime}^3}\left\{\frac{3}{4}\left\{1 + \frac{3}{2}e^2 + \frac{3}{2}e_{\prime}^2\right\} + \frac{3}{4}\left\{1 - \frac{5}{2}e^2 - \frac{5}{2}e_{\prime}^2\right\}\cos 2t - \frac{3}{2}e\cos x\right.$$

$$- \frac{9}{4}e\cos(2t - x) + \frac{3}{4}e\cos(2t + x) + \frac{9}{4}e_{\prime}\cos z + \frac{21}{8}e_{\prime}\cos(2t - z)$$

$$- \frac{3}{8}e_{\prime}\cos(2t + z) - \frac{3}{8}e^2\cos 2x + \frac{15}{8}e^2\cos(2t - 2x)$$

$$+ \frac{3}{4}e^2\cos(2t + 2x) - \frac{9}{4}e e_{\prime}\cos(x + z) - \frac{63}{8}e e_{\prime}\cos(2t - x - z)$$

$$- \frac{3}{8}e e_{\prime}\cos(2t + x + z) - \frac{9}{4}e e_{\prime}\cos(x - z) + \frac{9}{8}e e_{\prime}\cos(2t - x + z)$$

$$+ \frac{21}{8}e e_{\prime}\cos(2t + x - z) + \frac{27}{8}e_{\prime}^2\cos 2z + \frac{51}{8}e_{\prime}^2\cos(2t - 2z)$$

$$\frac{dR}{ds} = \frac{m_{\prime}a^2}{a_{\prime}^3}\left\{\underset{[146]}{\frac{204}{137}\gamma\sin y} - \underset{[147]}{\frac{20}{27}\gamma\sin(2t - y)} + \underset{[148]}{\frac{20}{27}\sin(2t + y)} + \underset{[149]}{\frac{3}{2}e\sin(x - y)}\right.$$

$$- \underset{[150]}{\frac{3}{2}e(\sin x + y)} + \underset{[151]}{\frac{9}{4}e\gamma\sin(2t - x - y)} - \underset{[152]}{\frac{9}{4}e\gamma\sin(2t - x + y)}$$

$$- \underset{[153]}{\frac{3}{4}e\gamma\sin(2t + x - y)} + \underset{[154]}{\frac{3}{4}e\gamma\sin(2t + x + y)} - \underset{[155]}{\frac{9}{4}e_{\prime}\gamma\sin(z - y)}$$

$$+ \underset{[156]}{\frac{9}{4}e_{\prime}\gamma\sin(z + y)} - \underset{[157]}{\frac{21}{8}e_{\prime}\gamma\sin(2t - z - y)} + \underset{[158]}{\frac{21}{8}e_{\prime}\gamma\sin(2t - z + y)}$$

$$+ \underset{[159]}{\frac{3}{8}e_{\prime}\gamma\sin(2t + z - y)} - \underset{[160]}{\frac{3}{8}e_{\prime}\sin(2t + z + y)} + \underset{[161]}{\frac{3}{8}e^2\gamma\sin(2x - y)}$$

$$- \underset{[162]}{\frac{3}{8}e^2\gamma\sin(2x + y)} - \underset{[163]}{\frac{15}{8}e^2\gamma\sin(2t - 2x - y)} + \underset{[164]}{\frac{15}{8}e^2\gamma\sin(2t - 2x + y)}$$

$$- \underset{[165]}{\frac{3}{4}e^2\gamma\sin(2t + 2x - y)} + \underset{(166)}{\frac{3}{4}e^2\gamma\sin(2t + 2x + y)}$$

$$+ \underset{[167]}{\frac{9}{4}e e_{\prime}\gamma\sin(x + z - y)} - \underset{[168]}{\frac{9}{4}e e_{\prime}\gamma\sin(x + z + y)}$$

* See Phil. Trans. 1831, p. 255 and 263.

$$+\frac{63}{8}ee_{\prime}\gamma\sin(2t-x-z-y)-\frac{63}{8}ee_{\prime}\gamma\sin(2t-x-z+y)$$
[169] [170]

$$+\frac{3}{8}ee_{\prime}\gamma\sin(2t+x+z-y)-\frac{3}{8}ee_{\prime}\gamma\sin(2t+x+z+y)$$
[171] [172]

$$+\frac{9}{4}ee_{\prime}\gamma\sin(x-z-y)-\frac{9}{4}ee_{\prime}\gamma\sin(x-z+y)$$
[173] [174]

$$-\frac{9}{8}ee_{\prime}\gamma\sin(2t-x+z-y)+\frac{9}{8}ee_{\prime}\gamma\sin(2t-x+z+y)$$
[175] [176]

$$-\frac{21}{8}ee_{\prime}\gamma\sin(2t+x-z-y)+\frac{21}{8}ee_{\prime}\gamma\sin(2t+x-z+y)$$
[177] [178]

$$-\frac{27}{8}e_{\prime}^2\gamma\sin(2z-y)+\frac{27}{8}e_{\prime}^2\gamma\sin(2z+y)-\frac{51}{8}e_{\prime}^2\gamma\sin(2t-2z-y)$$
[179] [180] [181]

$$+\frac{51}{8}e_{\prime}^2\gamma\sin(2t-2z+y)$$
[182]

The inequality of latitude of which the argument is $2t-y$ being far greater than the rest, $\delta s=\gamma s_{147}\sin(2t-y)$ nearly.

If $e=\cdot0548442$ $e_{\prime}=\cdot0167927$ $\gamma=\cdot0900684$

See Mém. sur la Théorie de la Lune, p. 502.

$$R=\frac{m_{\prime}a^2}{a_{\prime}^3}\Big\{-9\cdot3947865-9\cdot8697237\cos 2t+9\cdot6933013\,e\cos x$$
[0] [1] [2]

$$+0\cdot3494165\,e\cos(2t-x)-9\cdot8698883\,e\cos(2t+x)$$
[3] [4]

$$-9\cdot8718614\,e_{\prime}\cos z-9\cdot4138294\,e_{\prime}\cos(2t-z)$$
[5] [6]

$$+9\cdot5685221\,e_{\prime}\cos(2t+z)+9\cdot0917777\,e^2\cos 2x$$
[7] [8]

$$-0\cdot2709438\,e^2\cos(2t-2x)-9\cdot8697180\,e^2\cos(2t+2x)$$
[9] [10]

$$+9\cdot8697237\,ee_{\prime}\cos(x+z)+0\cdot8935219\,ee_{\prime}\cos(2t-x-z)$$
[11] [12]

$$+ 9{\cdot}5691515\, e\, e_{,} \cos(2t + x + z) + 9{\cdot}8697180\, e\, e_{,} \cos(x - z)$$
[13] [14]

$$- 0{\cdot}0486780\, e\, e_{,} \cos(2t - x + z) + 0{\cdot}4139940\, e\, e_{,} \cos(2t + x - z)$$
[15] [16]

$$- 0{\cdot}0479097\, e_{,}^2 \cos 2z - 0{\cdot}7991728\, e_{,}^2 \cos(2t - 2z)$$
[17] [18]

$$- 9{\cdot}5709386\, \gamma^2 \cos 2z - 9{\cdot}5761195\, \gamma^2 \cos(2t - 2y)$$
[62] [63]

where the logarithms of the coefficients are written instead of the coefficients themselves.

$$R = \frac{m_{,} a^2}{a_{,}^3}\Big\{ -\frac{34}{137} - \frac{20}{27}\cos 2t + \frac{38}{77} e \cos x + \frac{38}{17} e \cos(2t - x) - \frac{20}{27} e \cos(2t + z)$$
[0] [1] [2] [3] [4]

$$- \frac{32}{43} e_{,} \cos z - \frac{70}{27} e_{,} \cos(2t - z) + \frac{10}{27} e_{,} \cos(2t + z) + \frac{10}{81} e^2 \cos 2x$$
[5] [6] [7] [8]

$$- \frac{28}{15} e^2 \cos(2t - 2x) - \frac{20}{27} e^2 \cos(2t + 2x) + \frac{20}{27} e\, e_{,} \cos(x + z)$$
[9] [10] [11]

$$+ \frac{180}{23} e\, e_{,} \cos(2t - x - z) + \frac{10}{27} e\, e_{,} \cos(2t + x + z) + \frac{20}{27} e\, e_{,} \cos(x + z)$$
[12] [13] [14]

$$- \frac{66}{59} e\, e_{,} \cos(2t - x + z) - \frac{83}{32} e\, e_{,} \cos(2t + x - z) - \frac{67}{60} e_{,}^2 \cos 2z$$
[15] [16] [17]

$$- \frac{233}{37} e_{,}^2 \cos(2t - 2z) - \frac{16}{43}\gamma^2 \cos 2y - \frac{26}{69}\gamma^2 \cos(2t - 2y) \text{ nearly.}$$
[18] [62] [63]

I make use of these approximate coefficients in the following development solely in order that it may occupy less space.

$$\delta R^* = \frac{m_{,} a^2}{a_{,}^3}\left[\frac{68}{137} r_0'{}^\dagger + \frac{20}{27}\left\{r_1' + \lambda_1\right\} - \frac{38}{77} e^2 r_2' - \frac{38}{17} e^2 \left\{r_3' + \lambda_3\right\} + \frac{20}{27} e^2 \left\{r_4' + \lambda_4\right\}\right.$$

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* See Phil. Trans. 1831, p. 275.

† $r\,\delta\frac{1}{r} = r_0' + r_1' \cos 2t + e r_2' \cos x + e r_3' \cos(2t - x)$ &c.
[0] [1] [2] [3]

$\delta\lambda = \lambda_1 \sin 2t + e \lambda_3 \sin(2t - x) +$ &c.
[1] [3]

Development of δR.

$$+\frac{32}{43}e_{\prime}^{2}r_5'+\frac{70}{27}e_{\prime}^{2}\left\{r_6'+\lambda_6\right\}-\frac{10}{27}e_{\prime}^{2}\left\{r_7'+\lambda_7\right\}-\frac{10}{27}\gamma^2 s_{147}$$

[0]

$$+\left\{+\frac{40}{27}r_0'+\frac{68}{137}r_1'-\frac{38}{17}e^2r_2'+\frac{20}{27}e^2r_2'-\frac{38}{77}e^2r_3'-\frac{38}{77}e^2r_4'+\frac{70}{27}e_{\prime}^{2}\left\{r_5'-\lambda_5\right\}\right.$$

$$-\frac{10}{27}e_{\prime}^{2}\left\{r_5'+\lambda_5\right\}+\frac{32}{43}e_{\prime}^{2}r_6'+\frac{32}{43}e_{\prime}^{2}r_7'+\frac{28}{15}e^4\left\{r_8'-\lambda_8\right\}+\frac{20}{27}e^4\left\{r_8'+\lambda_8\right\}$$

$$-\frac{10}{81}e^4r_9'-\frac{10}{81}e^4r_{10}'-\frac{180}{23}e^2e_{\prime}^{2}\left\{r_{11}'-\lambda_{11}\right\}-\frac{10}{27}e^2e_{\prime}^{2}\left\{r_{11}'+\lambda_{11}\right\}$$

$$-\frac{20}{27}e^2e_{\prime}^{2}r_{12}'+\frac{66}{59}e^2e_{\prime}^{2}\left\{r_{14}'-\lambda_{14}\right\}+\frac{83}{32}e^2e_{\prime}^{2}\left\{r_{14}'+\lambda_{14}\right\}$$

$$\left.-\frac{102}{137}\gamma^2 s_{147}\right\}\cos 2t$$

[1]

$$+\left\{-\frac{76}{77}r_0'+\frac{20}{27}\left\{r_1'+\lambda_1\right\}-\frac{38}{17}\left\{r_1'+\lambda_1\right\}+\frac{68}{137}r_2'+\frac{28}{15}e^2\left\{r_3'+\lambda_3\right\}\right.$$

$$+\frac{20}{27}\left\{r_3'+\lambda_3\right\}+\frac{20}{27}\left\{r_4'+\lambda_4\right\}-\frac{20}{27}e_{\prime}^{2}r_5'-\frac{20}{27}e_{\prime}^{2}r_5'-\frac{3}{8}\gamma^2 s_{147}$$

$$\left.+\frac{9}{8}\gamma^2 s_{147}\right\}e\cos x$$

[2]

$$+\left\{-\frac{76}{17}r_0'-\frac{38}{77}r_1'+\frac{20}{27}r_2'+\frac{68}{137}r_3'-\frac{180}{23}e_{\prime}^{2}\left\{r_5'-\lambda_5\right\}+\frac{66}{59}e_{\prime}^{2}\left\{r_5'+\lambda_5\right\}\right.$$

$$\left.+\frac{3}{4}\gamma^2 s_{147}\right\}e\cos(2t-x)$$

[3]

$$+\left\{+\frac{40}{27}r_0'-\frac{38}{77}r_1'+\frac{20}{27}r_2'-\frac{10}{81}e^2r_3'+\frac{68}{137}r_4'+\frac{83}{32}e_{\prime}^{2}\left\{r_5'-\lambda_5\right\}-\frac{10}{27}e_{\prime}^{2}\left\{r_5'+\lambda_5\right\}\right.$$

$$\left.+\frac{3}{4}\gamma^2 s_{147}\right\}e\cos(2t+x)$$

[4]

$$+\left\{+\frac{64}{43}r_0'-\frac{10}{27}\left\{r_1'+\lambda_1\right\}+\frac{70}{27}\left\{r_1'+\lambda_1\right\}+\frac{66}{59}e^2\left\{r_3'+\lambda_3\right\}-\frac{180}{23}e^2\left\{r_3'+\lambda_3\right\}\right.$$

$$\left.+\frac{68}{137}r_5'+\frac{67}{60}e_{\prime}^{2}r_5'-\frac{3}{16}\gamma^2 s_{147}-\frac{21}{16}\gamma^2 s_{147}\right\}e_{\prime}\cos z$$

[5]

$$+\left\{+\frac{140}{27}r_0'+\frac{32}{43}r_1'-\frac{20}{27}e^2r_3'+\frac{233}{37}e_{\prime}^{2}\left\{r_5'-\lambda_5\right\}+\frac{20}{27}\left\{r_5'+\lambda_5\right\}+\frac{68}{137}r_6'\right.$$

$$\left.-\frac{9}{8}\gamma^2 s_{147}\right\}e_{\prime}\cos(2t-z)$$

[6]

Development of δR.

$$+\left\{-\frac{20}{27}r_0'+\frac{32}{43}r_1'-\frac{20}{27}e^2r_3'+\frac{20}{27}\left\{r_5'-\lambda_5\right\}+\frac{68}{137}r_7'-\frac{9}{8}\gamma^2 s_{147}\right\}e_{\prime}\cos(2t+z) \quad [7]$$

$$+\left\{-\frac{20}{81}r_0'+\frac{20}{27}\left\{r_1'+\lambda_1\right\}+\frac{28}{15}\left\{r_1'+\lambda_1\right\}-\frac{7}{32}e^2\left\{r_3'+\lambda_3\right\}+\frac{20}{27}\left\{r_3'+\lambda_3\right\}\right.$$
$$-\frac{38}{17}\left\{r_4'+\lambda_4\right\}-\frac{3}{16}e_{\prime}^2r_5'-\frac{3}{16}e_{\prime}^2r_5'+\frac{68}{137}r_8'+\frac{20}{27}\left\{r_9'+\lambda_9\right\}$$
$$\left.+\frac{20}{27}\left\{r_{10}'+\lambda_{10}\right\}-\frac{3}{8}\gamma^2 s_{147}-\frac{15}{16}\gamma^2 s_{147}\right\}e^2\cos 2x \quad [8]$$

$$+\left\{+\frac{56}{15}r_0'-\frac{10}{81}r_1'-\frac{38}{17}r_2'-\frac{38}{77}r_3'+\frac{105}{16}e_{\prime}^2\left\{r_5'-\lambda_5\right\}-\frac{15}{16}e_{\prime}^2\left\{r_5'+\lambda_5\right\}\right.$$
$$\left.+\frac{20}{27}\left\{r_8'+\lambda_8\right\}+\frac{68}{137}r_9'+\frac{3}{16}\gamma^2 s_{147}\right\}e^2\cos(2t-2x) \quad [9]$$

$$+\left\{+\frac{40}{27}r_0'-\frac{10}{81}r_1'+\frac{20}{27}r_2'-\frac{1}{16}e^2r_3'-\frac{38}{77}r_4'+\frac{21}{8}e_{\prime}^2\left\{r_5'-\lambda_5\right\}-\frac{3}{8}e_{\prime}^2\left\{r_5'+\lambda_5\right\}\right.$$
$$\left.+\frac{20}{27}\left\{r_8'-\lambda_8\right\}+\frac{68}{137}r_{10}'+\frac{3}{16}\gamma^2 s_{147}\right\}e^2\cos(2t+2x) \quad [10]$$

$$+\left\{-\frac{40}{27}r_0'-\frac{10}{27}\left\{r_1'+\lambda_1\right\}-\frac{180}{23}\left\{r_1'+\lambda_1\right\}+\frac{105}{16}\left\{r_3'+\lambda_3\right\}-\frac{10}{27}\left\{r_3'+\lambda_3\right\}\right.$$
$$+\frac{70}{27}\left\{r_4'+\lambda_4\right\}-\frac{38}{77}r_5'-\frac{9}{8}e_{\prime}^2r_5'+\frac{20}{27}\left\{r_6'+\lambda_6\right\}-\frac{38}{17}\left\{r_7'+\lambda_7\right\}$$
$$-\frac{20}{27}e^2r_8'+\frac{66}{59}e^2\left\{r_9'+\lambda_9\right\}+\frac{83}{32}e^2\left\{r_{10}'+\lambda_{10}\right\}+\frac{68}{137}r_{11}'+\frac{20}{27}\left\{r_{12}'+\lambda_{12}\right\}$$
$$\left.-\frac{10}{81}e^2r_{14}'+\frac{3}{16}\gamma^2 s_{147}+\frac{63}{16}\gamma^2 s_{147}\right\}e\,e_{\prime}\cos(x+z) \quad [11]$$

$$+\left\{-\frac{360}{23}r_0'-\frac{20}{27}r_1'+\frac{70}{27}r_2'+\frac{32}{43}r_3'-\frac{153}{8}e_{\prime}^2\left\{r_5'-\lambda_5\right\}-\frac{38}{17}\left\{r_5'+\lambda_5\right\}-\frac{38}{77}r_6'\right.$$
$$\left.+\frac{20}{27}\left\{r_{11}'+\lambda_{11}\right\}+\frac{68}{137}r_{12}'+\frac{9}{8}\gamma^2 s_{147}\right\}e\,e_{\prime}\cos(2t-x-z) \quad [12]$$

$$+\left\{-\frac{20}{27}r_0'-\frac{20}{27}r_1'-\frac{10}{27}r_2'-\frac{3}{16}e^2\left\{r_3'-\lambda_3\right\}+\frac{32}{43}r_4'+\frac{20}{27}\left\{r_5'-\lambda_5\right\}-\frac{38}{77}r_7'\right.$$
$$\left.+\frac{20}{27}\left\{r_{11}'-\lambda_{11}\right\}+\frac{68}{137}r_{13}'+\frac{9}{8}\gamma^2 s_{147}\right\}e\,e_{\prime}\cos(2t+x+z) \quad [13]$$

Development of δR.

$$+\left\{-\frac{40}{27}r_0'+\frac{83}{32}\left\{r_1'+\lambda_1\right\}+\frac{66}{59}\left\{r_1'+\lambda_1\right\}+\frac{32}{43}r_2'-\frac{15}{16}e^2\left\{r_3'+\lambda_3\right\}+\frac{70}{27}\left\{r_3'+\lambda_3\right\}\right.$$

$$-\frac{10}{27}\left\{r_4'+\lambda_4\right\}-\frac{9}{8}e_{\prime}^2r_5'-\frac{38}{77}r_5'-\frac{38}{17}\left\{r_6'+\lambda_6\right\}+\frac{20}{27}\left\{r_7'+\lambda_7\right\}$$

$$-\frac{20}{27}e^2r_8-\frac{180}{23}e^2\left\{r_9'+\lambda_9\right\}-\frac{10}{27}e^2\left\{r_{10}'+\lambda_{10}\right\}-\frac{10}{81}e^2r_{11}'$$

$$+\frac{28}{15}e^2\left\{r_{12}'+\lambda_{12}\right\}+\frac{68}{137}r_{14}'+\frac{20}{27}\left\{r_{15}'+\lambda_{15}\right\}+\frac{20}{27}\left\{r_{16}'+\lambda_{16}\right\}$$

$$\left.-\frac{21}{16}\gamma^2s_{147}-\frac{9}{16}\gamma^2s_{147}\right\}ee_{\prime}\cos(x-z)$$

[14]

$$+\left\{+\frac{132}{59}r_0'-\frac{20}{27}r_1'-\frac{10}{27}r_2'+\frac{32}{43}r_3'-\frac{38}{17}\left\{r_5'-\lambda_5\right\}+\frac{20}{27}\left\{r_{14}'+\lambda_{14}\right\}\right.$$

$$\left.+\frac{68}{137}r_{15}'+\frac{9}{8}\gamma^2s_{147}\right\}ee_{\prime}\cos(2t-x+z)$$

[15]

$$+\left\{+\frac{83}{16}r_0'-\frac{20}{27}r_1'+\frac{70}{27}r_2'-\frac{3}{16}e^2r_3'+\frac{32}{43}r_4'+\frac{51}{8}e_{\prime}^2\left\{r_5'-\lambda_5\right\}+\frac{20}{27}\left\{r_5'+\lambda_5\right\}\right.$$

$$\left.-\frac{38}{77}r_6'+\frac{20}{27}\left\{r_{14}'-\lambda_{14}\right\}+\frac{68}{137}r_{16}'+\frac{9}{8}\gamma^2s_{147}\right\}ee_{\prime}\cos(2t+x-z)$$

[16]

$$+\left\{+\frac{67}{30}r_0'+\frac{233}{37}\left\{r_1'+\lambda_1\right\}-\frac{153}{8}e^2\left\{r_3'+\lambda_3\right\}+\frac{32}{43}r_5'+\frac{53}{32}e_{\prime}^2r_5'-\frac{10}{27}\left\{r_6'+\lambda_6\right\}\right.$$

$$+\frac{70}{27}\left\{r_7'+\lambda_7\right\}+\frac{68}{137}r_{17}+\frac{20}{27}\left\{r_{18}'+\lambda_{18}\right\}+\frac{20}{27}\left\{r_{19}'+\lambda_{19}\right\}$$

$$\left.-\frac{51}{16}\gamma^2s_{147}\right\}e_{\prime}^2\cos 2z$$

[17]

$$+\left\{+\frac{466}{37}r_0'+\frac{67}{60}r_1'-\frac{9}{8}e^2r_3'+\frac{845}{64}e_{\prime}^2\left\{r_5'-\lambda_5\right\}+\frac{70}{27}\left\{r_5'+\lambda_5\right\}+\frac{32}{43}r_6'\right.$$

$$\left.+\frac{20}{27}\left\{r_{17}'+\lambda_{17}\right\}+\frac{68}{137}r_{18}'-\frac{27}{16}\gamma^2s_{147}\right\}e_{\prime}^2\cos(2t-2z)$$

[18]

$$+\left\{+\frac{67}{60}r_1'-\frac{9}{8}e^2r_3'-\frac{10}{27}\left\{r_5'-\lambda_5\right\}+\frac{e_{\prime}^2}{64}\left\{r_5'+\lambda_5\right\}+\frac{32}{43}r_7'+\frac{20}{27}\left\{r_{17}'-\lambda_{17}\right\}\right.$$

$$\left.+\frac{68}{137}r_{19}'-\frac{27}{16}\gamma^2s_{147}\right\}e_{\prime}^2\cos(2t+2z)$$

[19]

Development of δR.

$$+\left\{+\frac{26}{69}\left\{r_1'+\lambda_1\right\}-\frac{3}{8}e^2\left\{r_3'+\lambda_3\right\}+\frac{9}{16}e_{\prime}^2 r_5'+\frac{9}{16}e_{\prime}^2 r_5'+\frac{10}{27}s_{147}\right\}\gamma^2\cos 2y$$
[62]

$$+\left\{+\frac{16}{43}r_1'-\frac{9}{8}r_3'+\frac{21}{16}e_{\prime}^2\left\{r_5'-\lambda_5\right\}-\frac{3}{16}e_{\prime}^2\left\{r_5'+\lambda_5\right\}+\frac{102}{137}s_{147}\right\}\gamma^2\cos(2t-2y)$$
[63]

$$+\left\{+\frac{16}{43}r_1'+\frac{3}{8}r_3'\right\}\gamma^2\cos(2t+2y)$$
[64]

$$+\left\{-\frac{3}{8}\left\{r_1'+\lambda_1\right\}+\frac{26}{69}\left\{r_3'+\lambda_3\right\}-\frac{9}{8}s_{147}\right\}\gamma^2 e\cos(x-2y)$$
[65]

$$+\left\{-\frac{3}{8}\left\{r_1'+\lambda_1\right\}+\frac{3}{8}s_{147}\right\}\gamma^2 e\cos(x+2y)$$
[66]

$$+\left\{+\frac{3}{8}r_1'+\frac{16}{43}r_3'-\frac{3}{4}s_{147}\right\}\gamma^2 e\cos(2t-x-2y)$$
[67]

$$+\left\{-\frac{9}{8}r_1'+\frac{16}{43}r'_3\right\}\gamma^2 e\cos(2t-x+2y)$$
[68]

$$+\left\{-\frac{9}{8}r_1'-\frac{3}{4}s_{147}\right\}\gamma^2 e\cos(2t+x-2y)+\frac{3}{8}r_1'\gamma^2 e\cos(2t+x+2y)$$
[69] [70]

$$+\left\{-\frac{3}{16}\{r_1'+\lambda_1\}+\frac{16}{43}r'_5+\frac{21}{16}s_{147}\right\}\gamma^2 e_{\prime}\cos(z-2y)$$
[71]

$$+\left\{+\frac{21}{16}\{r_1'+\lambda_1\}+\frac{16}{43}r_5'+\frac{3}{16}s_{147}\right\}\gamma^2 e_{\prime}\cos(z+2y)$$
[72]

$$+\left\{+\frac{9}{16}r_1'+\frac{26}{69}r_5'+\frac{9}{8}s_{147}\right\}\gamma^2 e_{\prime}\cos(2t-z-2y)$$
[73]

$$+\frac{9}{16}r_1'\gamma^2 e_{\prime}\cos(2t-z+2y)$$
[74]

$$+\left\{+\frac{9}{16}r_1'+\frac{26}{69}\{r_5'-\lambda_5\}+\frac{9}{8}s_{147}\right\}\gamma^2 e_{\prime}\cos(2t+z-2y)$$
[75]

$$+\frac{9}{16}r_1'\gamma^2 e_{\prime}\cos(2t+z+2y)$$
[76]

Development of δR.

$$+\left\{-\frac{3}{8}\{r_3'+\lambda_3\}+\frac{15}{16}s_{147}\right\}\gamma^2 e^2\cos(2x-2y)$$
[77]

$$+\frac{3}{8}s_{147}\gamma^2 e^2\cos(2x+2y)+\left\{+\frac{3}{8}r_3'-\frac{3}{16}s_{147}\right\}\gamma^2 e^2\cos(2t-2x-2y)$$
[78] [79]

$$-\frac{9}{8}r_3'\gamma^2 e^2\cos(2t-2x+2y)-\frac{3}{16}s_{147}\gamma^2 e^2\cos(2t+2x-2y)$$
[80] [81]

$$+\left\{-\frac{3}{16}\{r_3'+\lambda_3\}-\frac{9}{8}r_5'-\frac{63}{16}s_{147}'\right\}\gamma^2 e e_, \cos(x+z-2y)$$
[83]

$$+\left\{+\frac{3}{8}r_5'-\frac{3}{16}s_{147}\right\}\gamma^2 e e_, \cos(x+z+2y)$$
[84]

$$+\left\{+\frac{9}{16}r_3'-\frac{3}{8}\{r_5'+\lambda_5\}-\frac{9}{8}s_{147}\right\}\gamma^2 e e_, \cos(2t-x-z-2y)$$
[85]

$$+\frac{9}{16}r_3'\gamma^2 e e_, \cos(2t-x-z+2y)$$
[86]

$$+\left\{-\frac{3}{8}\{r_5'-\lambda_5\}-\frac{9}{8}s_{147}\right\}\gamma^2 e e_, \cos(2t+x+z-2y)$$
[87]

$$+\left\{+\frac{21}{16}\{r_3'+\lambda_3\}-\frac{9}{8}r_5'+\frac{9}{16}s_{147}\right\}\gamma^2 e e_, \cos(x-z-2y)$$
[89]

$$+\left\{+\frac{3}{8}r_5'+\frac{21}{16}s_{147}\right\}\gamma^2 e e_, \cos(x-z+2y)$$
[90]

$$+\left\{+\frac{9}{16}r_3'-\frac{3}{8}\{r_5'-\lambda_5\}-\frac{9}{8}s_{147}\right\}\gamma^2 e e_, \cos(2t-x+z-2y)$$
[91]

$$+\frac{9}{16}r_3'\gamma^2 e e_, \cos(2t-x+z+2y)$$
[92]

$$+\left\{-\frac{3}{8}\{r_5'+\lambda_5\}-\frac{9}{8}s_{147}\right\}\gamma^2 e e_, \cos(2t+x-z-2y)$$
[93]

$$+\left\{+\frac{9}{16}r_5'+\frac{51}{16}s_{147}\right\}\gamma^2 e_,^2\cos(2z-2y)+\frac{9}{16}r_5'\gamma^2 e_,^2\cos(2z-2y)$$
[95] [96]

Development of δR.

$$+\frac{a}{a_{\prime}}\Big\{+\frac{5.3}{8.2}\{r_1'+\lambda_1\}+\frac{3}{8}\Big\{\frac{3}{2}r_1'+\frac{1}{2}\lambda_1\Big\}-\frac{45.3}{16.2}e^2\{r_3'+\lambda_3\}$$
$$-\frac{15}{16}e^2\Big\{\frac{3}{2}r_3'+\frac{1}{2}\lambda_3\Big\}+\frac{9}{8}e_{\prime}^2\Big\{\frac{3}{2}r_5'-\frac{1}{2}\lambda_5\Big\}+\frac{3}{8}e_{\prime}^2\Big\{\frac{3}{2}r_5'+\frac{1}{2}\lambda_5\Big\}\Big\}\cos t^* \quad [101]$$

$$+\frac{a}{a_{\prime}}\Big\{-\frac{45.3}{16.2}\{r_1'+\lambda_1\}-\frac{3}{16}\Big\{\frac{3}{2}r_1'+\frac{1}{2}\lambda_1\Big\}+\frac{3}{8}\Big\{\frac{3}{2}r_3'+\frac{1}{2}\lambda_3\Big\}\Big\}e\cos(t-x) \quad [102]$$

$$+\frac{a}{a_{\prime}}\Big\{+\frac{15.3}{16.2}\{r_1'+\lambda_1\}-\frac{15}{16}\Big\{\frac{3}{2}r_1'+\frac{1}{2}\lambda_1\Big\}+\frac{5.3}{8.2}\{r_3'+\lambda_3\}\Big\}e\cos(t+x) \quad [103]$$

$$+\frac{a}{a_{\prime}}\Big\{+\frac{25.3}{8.2}\{r_1'+\lambda_1\}+\frac{3}{8}\Big\{\frac{3}{2}r_1'+\frac{1}{2}\lambda_1\Big\}+\frac{3}{8}\Big\{\frac{3}{2}r_5'+\frac{1}{2}\lambda_5\Big\}\Big\}e_{\prime}\cos(t-z) \quad [104]$$

$$+\frac{a}{a_{\prime}}\Big\{-\frac{5.3}{8.2}\{r_1'+\lambda_1\}+\frac{9}{8}\Big\{\frac{3}{2}r_1'+\frac{1}{2}\lambda_1\Big\}+\frac{3}{8}\Big\{\frac{3}{2}r_5'-\frac{1}{2}\lambda_5\Big\}\Big\}e_{\prime}\cos(t+z) \quad [105]$$

$$+\frac{a}{a_{\prime}}\Big\{-\frac{3}{16}\Big\{\frac{3}{2}r_3'+\frac{1}{2}\lambda_3\Big\}\Big\}e^2\cos(t-2x) \quad [106] \qquad +\frac{15.3}{16.2}\frac{a}{a_{\prime}}\{r_3'+\lambda_3\}e^2\cos(t+2x) \quad [107]$$

$$+\frac{a}{a_{\prime}}\Big\{+\frac{3}{8}\Big\{\frac{3}{2}r_3'+\frac{1}{2}\lambda_3\Big\}-\frac{15}{16}\Big\{\frac{3}{2}r_5'+\frac{1}{2}\lambda_5\Big\}\Big\}e\,e_{\prime}\cos(t-x-z) \quad [108]$$

$$+\frac{a}{a_{\prime}}\Big\{-\frac{5.3}{8.2}\{r_3'+\lambda_3\}-\frac{3}{16}\Big\{\frac{3}{2}r_5'-\frac{1}{2}\lambda_5\Big\}\Big\}e\,e_{\prime}\cos(t+x+z) \quad [109]$$

$$+\frac{a}{a_{\prime}}\Big\{+\frac{9}{8}\Big\{\frac{3}{2}r_3'+\frac{1}{2}\lambda_3\Big\}-\frac{15}{16}\Big\{\frac{3}{2}r_5'-\frac{1}{2}\lambda_5\Big\}\Big\}e\,e_{\prime}\cos(t-x+z) \quad [110]$$

$$+\frac{a}{a_{\prime}}\Big\{+\frac{25.3}{8.2}\{r_3'+\lambda_3\}-\frac{3}{16}\Big\{\frac{3}{2}r_5'+\frac{1}{2}\lambda_5\Big\}\Big\}e\,e_{\prime}\cos(t+x-z) \quad [111]$$

$$+\frac{a}{a_{\prime}}\frac{9}{8}\Big\{\frac{3}{2}r_5'+\frac{1}{2}\lambda_5\Big\}e_{\prime}^2\cos(t-2z) \quad [112] \qquad +\frac{a}{a_{\prime}}\frac{3}{8}\Big\{\frac{3}{2}r_5'-\frac{1}{2}\lambda_5\Big\}e_{\prime}^2\cos(t+2z) \quad [113]$$

$$+\frac{a}{a_{\prime}}\Big\{+\frac{3}{8}\Big\{\frac{3}{2}r_1'-\frac{1}{2}\lambda_1\Big\}-\frac{3}{16}\Big\{\frac{3}{2}r_3'-\frac{1}{2}\lambda_3\Big\}+\frac{25.3}{8.2}\{r_5'-\lambda_5\}$$
$$-\frac{5.3}{8.2}\{r_5'+\lambda_5\}\Big\}\cos 3t \quad [116]$$

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* In this development of δR the terms multiplied by $\frac{a^3}{a_{\prime}^4}\gamma^2 s_{147}$ are neglected.

Development of δR.

$$+\frac{a}{a_{\prime}}\left\{-\frac{15}{16}\left\{\frac{3}{2}r_1'-\frac{1}{2}\lambda_1\right\}+\frac{3}{8}\left\{\frac{3}{2}r_3'-\frac{1}{2}\lambda_3\right\}\right\}e\cos(3t-x)$$
[117]

$$-\frac{a}{a_{\prime}}\frac{3}{16}\left\{\frac{3}{2}r_1'-\frac{1}{2}\lambda_1\right\}e\cos(3t+x)$$
[118]

$$+\frac{a}{a_{\prime}}\left\{+\frac{9}{8}\left\{\frac{3}{2}r_1'-\frac{1}{2}\lambda_1\right\}+\frac{5.3}{8.2}\left\{r_1'+\lambda_1\right\}\right\}e_{\prime}\cos(3t-z)$$
[119]

$$+\frac{a}{a_{\prime}}\left\{+\frac{3}{8}\left\{\frac{3}{2}r_1'-\frac{1}{2}\lambda_1\right\}+\frac{5.3}{8.2}\left\{r_3'-\lambda_3\right\}\right\}e_{\prime}\cos(3t+z)$$
[120]

$$-\frac{a}{a_{\prime}}\frac{15}{16}\left\{\frac{3}{2}r_3'-\frac{1}{2}\lambda_3\right\}e^2\cos(3t-2x)$$
[121]

$$+\frac{a}{a_{\prime}}\left\{+\frac{9}{8}\left\{\frac{3}{2}r_3'-\frac{1}{2}\lambda_3\right\}-\frac{45.3}{16.2}\left\{r_5'+\lambda_5\right\}\right\}e\,e_{\prime}\cos(3t-x-z)$$
[123]

$$+\frac{a}{a_{\prime}}\frac{15.3}{16.2}\left\{r_5'-\lambda_5\right\}e\,e_{\prime}\cos(3t+x+z)$$
[124]

$$+\frac{a}{a_{\prime}}\left\{+\frac{3}{8}\left\{\frac{3}{2}r_3'-\frac{1}{2}\lambda_3\right\}-\frac{45.3}{16.2}\left\{r_5-\lambda_5\right\}\right\}e\,e_{\prime}\cos(3t-x+z)$$
[125]

$$+\frac{a}{a_{\prime}}\frac{15.3}{16.2}\left\{r_5'+\lambda_5\right\}e\,e_{\prime}\cos(3t+x-z)+\frac{25.3}{8.2}\left\{r_5'+\lambda_5\right\}e_{\prime}^2\cos(3t-2z)$$
[126] [127]

$$-\frac{5.3}{8.2}\left\{r_5'-\lambda_5\right\}e_{\prime}^2\cos(3t+2z)$$
[128]

$$+\left\{+\frac{20}{27}\left\{r_1'-\lambda_1\right\}+\frac{20}{27}e^2\left\{r_3'-\lambda_3\right\}-\frac{10}{27}\gamma^2 s_{147}\right\}\cos 4t$$
[131]

$$+\left\{-\frac{38}{17}\left\{r_1'-\lambda_1\right\}+\frac{20}{27}\left\{r_3'-\lambda_3\right\}+\frac{9}{8}\gamma^2 s_{147}\right\}e\cos(4t-x)$$
[132]

$$+\left\{+\frac{20}{27}\left\{r_1'-\lambda_1\right\}+\frac{20}{27}e^2\left\{r_3'-\lambda_3\right\}+\frac{20}{27}\left\{r_4'-\lambda_4\right\}-\frac{3}{8}\gamma^2 s_{147}\right\}e\cos(4t+x)$$
[133]

Development of δR.

$$+\left\{+\frac{70}{27}\left\{r_1'-\lambda_1\right\}+\frac{83}{32}e^2\left\{r_3'-\lambda_3\right\}-\frac{21}{16}\gamma^2 s_{147}\right\}e_{\prime}\cos(4t-z)$$

[134]

$$+\left\{-\frac{10}{27}\left\{r_1'-\lambda_1\right\}-\frac{10}{27}\left\{r_3'-\lambda_3\right\}-\frac{3}{16}\gamma^2 s_{147}\right\}e_{\prime}\cos(4t+z)$$

[135]

$$+\left\{+\frac{28}{15}\left\{r_1'-\lambda_1\right\}-\frac{38}{17}\left\{r_3'-\lambda_3\right\}-\frac{15}{16}\gamma^2 s_{147}\right\}e^2\cos(4t-2x)$$

[136]

$$+\left\{+\frac{20}{27}r_1'+\frac{20}{27}\left\{r_4'-\lambda_4\right\}-\frac{3}{8}s_{147}\right\}e^2\cos(4t+2x)$$

[137]

$$+\left\{-\frac{180}{23}\left\{r_1'-\lambda_1\right\}+\frac{70}{27}\left\{r_3'-\lambda_3\right\}+\frac{63}{16}\gamma^2 s_{147}\right\}e\,e_{\prime}\cos(4t-x-z)$$

[138]

$$+\left\{-\frac{10}{27}\left\{r_1'-\lambda_1\right\}-\frac{10}{27}\left\{r_4'-\lambda_4\right\}+\frac{3}{16}\gamma^2 s_{147}\right\}e\,e_{\prime}\cos(4t+x+z)$$

[139]

$$+\left\{+\frac{66}{59}\left\{r_1'-\lambda_1\right\}-\frac{10}{27}\left\{r_3'-\lambda_3\right\}-\frac{9}{16}\gamma^2 s_{147}\right\}e\,e_{\prime}\cos(4t-x+z)$$

[140]

$$+\left\{+\frac{83}{32}\left\{r_1'-\lambda_1\right\}+\frac{70}{27}\left\{r_4'-\lambda_4\right\}-\frac{21}{16}\gamma^2 s_{147}\right\}e\,e_{\prime}\cos(4t+x-z)$$

[141]

$$+\left\{+\frac{233}{37}\left\{r_1'-\lambda_1\right\}-\frac{51}{16}\gamma^2 s_{147}\right\}e_{\prime}^2\cos(4t-2z)$$

[142]

$\delta\,.\,\mathrm{d}R =$ the differential of δR, supposing only nt variable

$$+\frac{m^* m_{\prime} a^2}{a_{\prime}^3}\left\{\left\{+\frac{2.68}{137}r_1'-\frac{2.38}{77}e^2 r_3'-\frac{2.38}{77}e^2 r_4'-\frac{70}{27}e_{\prime}^2\left\{r_5'-\lambda_5\right\}-\frac{10}{27}e_{\prime}^2\left\{r_5'+\lambda_5\right\}\right.\right.$$

$$\left.+\frac{3.32}{43}e_{\prime}^2 r_6'+\frac{32}{43}e_{\prime}^2 r_7'-\frac{2.102}{137}\gamma^2 s_{147}\right\}\sin 2t$$

[1]

$$+\left\{-\frac{2.20}{27}\left\{r_1'+\lambda_1\right\}-\frac{2.38}{17}\left\{r_1'+\lambda_1\right\}+\frac{2.28}{15}e^2\left\{r_3'+\lambda_3\right\}\right.$$

$$-\frac{2.20}{27}\left\{r_3'+\lambda_3\right\}+\frac{2.20}{27}\left\{r_4'+\lambda_4\right\}+\frac{20}{27}e_{\prime}^2 r_5'-\frac{20}{27}e_{\prime}^2 r_5'-\frac{2.3}{8}\gamma^2 s_{147}$$

$$\left.-\frac{2.9}{8}\gamma^2 s_{147}\right\}e\sin x$$

[2]

* $m=\frac{n_{\prime}}{n}$ as in the notation of M. Damoiseau.

Development of $\delta\, d\, R$.

$$+\left\{-\frac{2.38}{77}r_1'+\frac{2.68}{137}r_3'+\frac{180}{23}e_{\prime}^2\left\{r_5'-\lambda_5\right\}+\frac{66}{59}e_{\prime}^2\left\{r_5'+\lambda_5\right\}-\frac{2.3}{4}\gamma^2 s_{147}\right\}e\sin(2t-x)$$

[3]

$$+\left\{-\frac{2.38}{77}r_1'-\frac{2.10}{81}e^2r_3'+\frac{2.68}{137}r_4'-\frac{83}{32}e_{\prime}^2\left\{r_5'-\lambda_5\right\}-\frac{10}{27}e_{\prime}^2\left\{r_5'+\lambda_5\right\}+\frac{2.3}{4}\gamma^2 s_{147}\right\}e\sin(2t+x)$$

[4]

$$+\left\{+\frac{2.10}{27}\left\{r_1'+\lambda_1\right\}+\frac{2.70}{27}\left\{r_1'+\lambda_1\right\}-\frac{2.66}{59}e^2\left\{r_3'+\lambda_3\right\}-\frac{2.180}{23}e^2\left\{r_3'+\lambda_3\right\}-\frac{68}{137}r_5'+\frac{67}{60}e_{\prime}^2r_5'-\frac{2.3}{16}\gamma^2 s_{147}+\frac{2.21}{16}\gamma^2 s_{147}\right\}e_{\prime}\sin z$$

[5]

$$+\left\{+\frac{2.32}{43}r_1'-\frac{2.20}{27}e^2r_3'-\frac{233}{37}e_{\prime}^2\left\{r_5'-\lambda_5\right\}+\frac{20}{27}\left\{r_5'+\lambda_5\right\}+\frac{3.68}{137}r_6'+\frac{2.9}{8}\gamma^2 s_{147}\right\}e_{\prime}\sin(2t-z)$$

[6]

$$+\left\{+\frac{2.32}{43}r_1'-\frac{2.20}{27}e^2r_3'-\frac{20}{27}\left\{r_5'-\lambda_5\right\}+\frac{68}{137}r_7'-\frac{2.9}{8}\gamma^2 s_{147}\right\}e_{\prime}\sin(2t+z)$$

[7]

$$+\left\{-\frac{2.20}{27}\left\{r_1'+\lambda_1\right\}+\frac{2.28}{15}\left\{r_1'+\lambda_1\right\}-\frac{2.7}{32}e^2\left\{r_3'+\lambda_3\right\}-\frac{2.20}{27}\left\{r_3'+\lambda_3\right\}-\frac{2.38}{17}\left\{r_4'+\lambda_4\right\}+\frac{3}{16}e_{\prime}^2r_5'-\frac{3}{16}e_{\prime}^2r_5-\frac{2.20}{27}\left\{r_9'+\lambda_9\right\}+\frac{2.20}{27}\left\{r_{10}+\lambda_{10}\right\}-\frac{2.3}{8}\gamma^2 s_{147}+\frac{2.15}{16}\gamma^2 s_{147}\right\}e^2\sin 2x$$

[8]

$$+\left\{-\frac{2.10}{81}r_1'-\frac{2.38}{77}r_3'-\frac{105}{16}e_{\prime}^2\left\{r_5'-\lambda_5\right\}-\frac{15}{16}e_{\prime}^2\left\{r_5'+\lambda_5\right\}+\frac{2.68}{137}r_9'-\frac{2.3}{16}\gamma^2 s_{147}\right\}e^2\sin(2t-2x)$$

[9]

$$+\left\{-\frac{2.10}{81}r_1'-\frac{2}{16}e^2r_3'-\frac{2.38}{77}r_4'-\frac{21}{8}e_{\prime}^2\left\{r_5'-\lambda_5\right\}-\frac{3}{8}e_{\prime}^2\left\{r_5'+\lambda_5\right\}+\frac{2.68}{137}r_{10}'\right.$$

Development of $\delta\, d\, R$.

$$+\frac{2.3}{16}\gamma^2 s_{147}\Big\} e^2 \sin(2t+2x)$$

[10]

$$+\Big\{+\frac{2.10}{27}\Big\{r_1'+\lambda_1\Big\}-\frac{2.180}{23}\Big\{r_1'+\lambda_1\Big\}+\frac{2.105}{16}e^2\Big\{r_3'+\lambda_3\Big\}+\frac{2.10}{27}\Big\{r_3'+\lambda_3\Big\}$$

$$+\frac{2.70}{27}\Big\{r_4'+\lambda_4\Big\}+\frac{38}{77}r_5'-\frac{9}{8}e_{\prime}^2 r_5'-\frac{3.20}{27}\Big\{r_6'+\lambda_6\Big\}-\frac{38}{17}\Big\{r_7+\lambda_7\Big\}$$

$$-\frac{2.66}{59}e^2\Big\{r_9'+\lambda_9\Big\}+\frac{2.83}{32}e^2\Big\{r_{10}'+\lambda_{10}\Big\}-\frac{68}{137}r_{11}'-\frac{3.20}{27}\Big\{r_{12}'+\lambda_{12}\Big\}$$

$$+\frac{3}{8}\gamma^2 s_{147}-\frac{2.63}{16}\gamma^2 s_{147}\Big\} e\, e_{\prime} \sin(x+z)$$

[11]

$$+\Big\{-\frac{2.20}{27}r_1'+\frac{2.32}{43}r_3'+\frac{153}{8}e_{\prime}^2\Big\{r_5'-\lambda_5\Big\}-\frac{38}{17}\Big\{r_5'+\lambda_5\Big\}-\frac{3.38}{77}r_6'$$

$$+\frac{20}{27}\Big\{r_{11}'+\lambda_{11}\Big\}+\frac{3.68}{137}r_{12}'-\frac{9}{4}\gamma^2 s_{147}\Big\} e\, e_{\prime} \sin(2t-x-z)$$

[12]

$$+\Big\{-\frac{2.20}{27}r_1'-\frac{2.3}{16}e^2\Big\{r_3'-\lambda_3\Big\}+\frac{2.32}{43}r_4'-\frac{20}{27}\Big\{r_5'-\lambda_5\Big\}-\frac{38}{77}r_7'$$

$$-\frac{20}{27}\Big\{r_{11}'-\lambda_{11}\Big\}+\frac{68}{137}r_{13}'+\frac{9}{4}\gamma^2 s_{147}\Big\} e\, e_{\prime} \sin(2t+x+z)$$

[13]

$$+\Big\{-\frac{2.83}{32}\Big\{r_1'+\lambda_1\Big\}+\frac{2.66}{59}\Big\{r_1'+\lambda_1\Big\}-\frac{2.15}{16}e^2\Big\{r_3'+\lambda_3\Big\}-\frac{2.70}{27}\Big\{r_3'+\lambda_3\Big\}$$

$$-\frac{2.10}{27}\Big\{r_4'+\lambda_4\Big\}+\frac{9}{8}e_{\prime}^2 r_5'-\frac{38}{77}e_{\prime}^2 r_5'-\frac{3.38}{77}\Big\{r_6'+\lambda_6\Big\}-\frac{20}{27}\Big\{r_7'+\lambda_7\Big\}$$

$$+\frac{68}{137}r_{14}'-\frac{2.21}{16}\gamma^2 s_{147}+\frac{2.9}{16}\gamma^2 s_{147}\Big\} e\, e_{\prime} \sin(x-z)$$

[14]

$$+\Big\{-\frac{2.20}{27}r_1'+\frac{2.32}{43}r_3'+\frac{38}{17}\Big\{r_5'-\lambda_5\Big\}-\frac{20}{27}\Big\{r_{14}'+\lambda_{14}\Big\}+\frac{68}{137}r_{15}'$$

$$-\frac{2.9}{8}\gamma^2 s_{147}\Big\} e\, e_{\prime} \sin(2t-x+z)$$

[15]

$$+\Big\{-\frac{2.20}{27}r_1'-\frac{2.3}{16}e^2 r_3'+\frac{2.32}{43}r_4'-\frac{51}{8}e_{\prime}^2\Big\{r_5'-\lambda_5\Big\}+\frac{20}{27}\Big\{r_5'+\lambda_5\Big\}-\frac{3.38}{77}r_6'$$

Development of $\delta\, d\, R$.

$$+\frac{20}{27}\left\{r_{14}'-\lambda_{14}\right\}+\frac{3.68}{137}r_{16}'+\frac{2.9}{8}\gamma^2 s_{147}\Big\}\, e\, e_{\prime} \sin(2t+x-z)$$ [16]

$$+\Big\{+\frac{2.233}{37}\left\{r_1'+\lambda_1\right\}-\frac{2.153}{8}e^2\left\{r_3'+\lambda_3\right\}-\frac{32}{43}r_5'+\frac{53}{32}e_{\prime}^2 r_5'+\frac{3.10}{27}\left\{r_6'+\lambda_6\right\}$$

$$+\frac{70}{27}\left\{r_7'+\lambda_7\right\}+\frac{2.51}{16}\gamma^2 s_{147}\Big\}e_{\prime}^2\sin 2z$$ [17]

$$+\Big\{+\frac{2.67}{60}r_1'-\frac{2.9}{8}e^2 r_3'-\frac{845}{64}e_{\prime}^2\left\{r_5'-\lambda_5\right\}+\frac{70}{27}\left\{r_5'+\lambda_5\right\}+\frac{3.32}{43}r_6'$$

$$+\frac{2.20}{27}\left\{r_{17}'+\lambda_{17}\right\}+\frac{4.68}{137}r_{18}'+\frac{2.27}{16}\gamma^2 s_{147}\Big\}\, e_{\prime}^2\sin(2t-2z)$$ [18]

$$+\Big\{+\frac{2.67}{60}r_1'-\frac{2.9}{8}e^2 r_3'+\frac{10}{27}\left\{r_5'-\lambda_5\right\}+\frac{e_{\prime}^2}{64}\left\{r_5+\lambda_5\right\}+\frac{32}{43}r_7'$$

$$-\frac{2.20}{17}\left\{r_{17}'-\lambda_{17}\right\}-\frac{2.27}{16}\gamma^2 s_{147}\Big\}\, e_{\prime}^2\sin(2t+2z)$$ [19]

$$+\Big\{+\frac{2.26}{69}\left\{r_1'+\lambda_1\right\}-\frac{2.3}{8}e^2\left\{r_3'+\lambda_3\right\}+\frac{9}{16}e_{\prime}^2 r_5'-\frac{9}{16}e_{\prime}^2 r_5'+\frac{20}{27}s_{147}\Big\}\gamma^2\cos 2y$$ [62]

$$+\Big\{+\frac{2.16}{43}r_1'-\frac{2.9}{8}e^2 r_3'-\frac{21}{16}e_{\prime}^2\left\{r_5'+\lambda_5\right\}-\frac{3}{16}e_{\prime}^2\left\{r_5'+\lambda_5\right\}$$

$$-\frac{102}{137}s_{147}\Big\}\gamma^2\cos(2t-2y)$$ [63]

$$+\Big\{+\frac{2.16}{43}r_1'+\frac{2.3}{8}e^2 r_3'\Big\}\gamma^2\cos(2t+2y)$$ [64]

$$+\Big\{+\frac{2.3}{8}\left\{r_1'+\lambda_1\right\}-\frac{2.26}{69}\left\{r_3'+\lambda_3\right\}+\frac{9}{4}s_{147}\Big\}\gamma^2 e\cos(x-2y)$$ [65]

$$+\Big\{-\frac{2.3}{8}\left\{r_1'+\lambda_1\right\}+\frac{3}{4}s_{147}\Big\}\gamma^2 e\cos(x+2y)$$ [66]

$$+\Big\{+\frac{2.3}{8}r_1'+\frac{2.16}{43}r_3'+\frac{3}{2}s_{147}\Big\}\gamma^2 e\cos(2t-x-2y)$$ [67]

Development of $\delta \, d R$.

$$+\left\{-\frac{2.9}{8}r_1'+\frac{2.16}{43}r_3'\right\}\gamma^2 e\cos(2t-x+2y)$$
[68]

$$+\left\{-\frac{2.9}{8}r_1'-\frac{3}{2}s_{147}\right\}\gamma^2 e\cos(2t+x-2y)+\frac{2.3}{8}r_1'\gamma^2 e\cos(2t+x+2y)$$
[69] [70]

$$+\left\{+\frac{2.3}{16}\left\{r_1'+\lambda_1\right\}-\frac{16}{43}r_5'-\frac{21}{8}s_{147}\right\}\gamma^2 e\cos(z-2y)$$
[71]

$$+\left\{+\frac{2.21}{16}\left\{r_1'+\lambda_1\right\}-\frac{16}{43}r_5'+\frac{3}{8}s_{147}\right\}\gamma^2 e_{,}\cos(z+2y)$$
[72]

$$+\left\{+\frac{2.9}{16}r_1'+\frac{26}{69}r_5'-\frac{9}{4}s_{147}\right\}\gamma^2 e_{,}\cos(2t-z-2y)+\frac{2.9}{16}r_1'\gamma^2 e_{,}\cos(2t-z+2y)$$
[73] [74]

$$+\left\{+\frac{2.9}{16}r_1'-\frac{26}{69}\left\{r_5'+\lambda_5\right\}+\frac{9}{4}s_{147}\right\}\gamma^2 e_{,}\cos(2t+z-2y)$$
[75]

$$+\frac{2.9}{16}r_1\gamma^2 e_{,}\cos(2t+z+2y)$$
[76]

$$+\frac{a}{a_{,}}\left\{-\frac{2.5.3}{8.2}\left\{r_1'+\lambda_1\right\}+\frac{2.3}{8}\left\{\frac{3}{2}r_1'+\frac{1}{2}\lambda_1\right\}+\frac{2.45.3}{16.2}e^2\left\{r_3'+\lambda_3\right\}\right.$$
$$-\frac{2.15}{16}e^2\left\{\frac{3}{9}r_3'+\frac{1}{2}\lambda_3\right\}-\frac{9}{8}e_{,}^2\left\{\frac{3}{2}r_5'+\frac{1}{2}\lambda_5\right\}$$
$$\left.+\frac{3}{8}e_{,}^2\left\{\frac{3}{2}r_5'+\frac{1}{2}\lambda_5\right\}\right\}\cos t$$
[101]

$$+\frac{a}{a_{,}}\left\{+\frac{2.45.3}{16.2}\left\{r_1'+\lambda_1\right\}-\frac{2.3}{16}\left\{\frac{3}{2}r_1'+\frac{1}{2}\lambda_1\right\}\right.$$
$$\left.+\frac{2.3}{8}\left\{\frac{3}{2}r_3'+\frac{1}{2}\lambda_3\right\}\right\}e\cos(t-x)$$
[102]

$$+\frac{a}{a_{,}}\left\{-\frac{2.15.3}{16.2}\left\{r_1'+\lambda_1\right\}-\frac{2.15}{16}\left\{\frac{3}{2}r_1'+\frac{1}{2}\lambda_1\right\}\right.$$
$$\left.-\frac{2.5.3}{8.2}\left\{r_3'+\lambda_3\right\}\right\}e\cos(t+x)$$
[103]

$$+\frac{a}{a_{,}}\left\{-\frac{2.25.3}{8.2}\left\{r_1'+\lambda_1\right\}+\frac{2.3}{8}\left\{\frac{3}{2}r_1'+\frac{1}{2}\lambda_1\right\}\right.$$

Development of $\delta\, d\, R$.

$$+\frac{3}{8}\left\{\frac{3}{2}r_5'+\frac{1}{2}\lambda_5\right\}\Big\}e_{\prime}\cos(t-z)$$

[104]

$$+\frac{a}{a_{\prime}}\Big\{+\frac{2.5.3}{8.2}\left\{r_1'+\lambda_1\right\}+\frac{2.9}{8}\left\{\frac{3}{2}r_1'+\frac{1}{2}\lambda_1\right\}$$

$$-\frac{3}{8}\left\{\frac{3}{2}r_5'-\frac{1}{2}\lambda_5\right\}\Big\}e_{\prime}\cos(t+z)$$

[105]

$$+\frac{a}{a_{\prime}}\Big\{+\frac{2.9}{8}\left\{\frac{3}{2}r_3'+\frac{1}{2}\lambda_3\right\}+\frac{15}{16}\left\{\frac{3}{2}r_5'-\frac{1}{2}\lambda_5\right\}\Big\}e\,e_{\prime}\cos(t-x+z)$$

[110]

$$+\frac{a}{a_{\prime}}\Big\{+\frac{2.3}{8}\left\{\frac{3}{2}r_1'-\frac{1}{2}\lambda_1\right\}-\frac{2.3}{16}e^2\left\{\frac{3}{2}r_3'-\frac{1}{2}\lambda_3\right\}-\frac{25.3}{8.2}e_{\prime}^2\left\{r_5'-\lambda_5\right\}$$

$$-\frac{5.3}{8.2}e_{\prime}^2\left\{r_5'+\lambda_5\right\}\Big\}\cos 3t$$

[116]

$$+\frac{a}{a_{\prime}}\Big\{-\frac{2.15}{16}\left\{\frac{3}{2}r_1'-\frac{1}{2}\lambda_1\right\}+\frac{2.3}{8}\left\{\frac{3}{2}r_3-\frac{1}{2}\lambda_3\right\}\Big\}e\cos(3t-x)$$

[117]

$$-\frac{a}{a_{\prime}}\frac{2.3}{16}\left\{\frac{3}{2}r_1-\frac{1}{2}\lambda_1\right\}e\cos(3t+x)$$

[118]

$$+\frac{a}{a_{\prime}}\Big\{+\frac{2.9}{8}\left\{\frac{3}{2}r_1'-\frac{1}{2}\lambda_1\right\}+\frac{5.3}{8.2}\left\{r_5'+\lambda_5\right\}\Big\}e_{\prime}\cos(3t-z)$$

[119]

$$+\frac{a}{a_{\prime}}\Big\{+\frac{2.3}{8}\left\{\frac{3}{2}r_1-\frac{1}{2}\lambda_1\right\}-\frac{5.3}{8.2}\left\{r_5'-\lambda_5\right\}\Big\}e_{\prime}\cos(3t+z)$$

[120]

$$+\Big\{+\frac{2.20}{27}\left\{r_1'-\lambda_1\right\}+\frac{2.20}{27}e^2\left\{r_3'-\lambda_3\right\}-\frac{20}{27}\gamma^2 s_{147}\Big\}\cos 4t$$

[131]

$$+\Big\{-\frac{2.38}{17}\left\{r_1'-\lambda_1\right\}+\frac{2.20}{27}\left\{r_3'-\lambda_3\right\}+\frac{9}{4}\gamma^2 s_{147}\Big\}e\cos(4t-x)$$

[132]

$$+\Big\{+\frac{2.20}{27}\left\{r_1'-\lambda_1\right\}+\frac{2.20}{27}\left\{r_3'-\lambda_3\right\}+\frac{2.20}{27}\left\{r_4'-\lambda_4\right\}$$

$$-\frac{3}{4}\gamma^2 s_{147}\Big\}e\cos(4t+x)$$

[133]

$$+\Big\{+\frac{2.70}{27}\left\{r_1'-\lambda_1\right\}+\frac{2.83}{32}e^2\left\{r_3'-\lambda_3\right\}-\frac{21}{8}\gamma^2 s_{147}\Big\}e_{\prime}\cos(4t-z)$$

[134]

Development of $\delta\, \mathrm{d}\, R$.

$$+\left\{-\frac{2\,.\,10}{27}\left\{r_1'-\lambda_1\right\}-\frac{2\,.\,10}{27}\left\{r_3'-\lambda_3\right\}-\frac{3}{8}\gamma^2 s_{147}\right\}e_{\prime}\cos(4\,t+z)$$ [135]

$$+\left\{+\frac{2\,.\,28}{15}\left\{r_1'-\lambda_1\right\}-\frac{2\,.\,38}{17}\left\{r_3'-\lambda_3\right\}-\frac{15}{8}\gamma^2 s_{147}\right\}e^2\cos(4\,t-2\,x)$$ [136]

$$+\left\{+\frac{2\,.\,20}{27}r_1'+\frac{2\,.\,20}{27}\left\{r_4'-\lambda_4\right\}-\frac{3}{4}\gamma^2 s_{147}\right\}e^2\cos(4\,t+2\,x)$$ [137]

$$+\left\{-\frac{2\,.\,180}{23}\left\{r_1'-\lambda_1\right\}+\frac{2\,.\,70}{27}\left\{r_3'-\lambda_3\right\}-\frac{63}{8}\gamma^2 s_{147}\right\}e\,e_{\prime}\cos(4\,t-x-z)$$ [138]

$$+\left\{-\frac{2\,.\,10}{27}\left\{r_1'-\lambda_1\right\}-\frac{2\,.\,10}{27}\left\{r_4'-\lambda_4\right\}-\frac{3}{8}\gamma^2 s_{147}\right\}e\,e_{\prime}\cos(4\,t+x+z)$$ [139]

$$+\left\{+\frac{2\,.\,66}{59}\left\{r_1'-\lambda_1\right\}-\frac{2\,.\,10}{27}\left\{r_3'-\lambda_3\right\}-\frac{9}{8}\gamma^2 s_{147}\right\}e\,e_{\prime}\cos(4\,t-x+z)$$ [140]

$$+\left\{+\frac{2\,.\,83}{32}\left\{r_1'-\lambda_1\right\}+\frac{70}{27}\left\{r_4'-\lambda_4\right\}-\frac{21}{8}\gamma^2 s_{147}\right\}e\,e_{\prime}\cos(4\,t+x-z)$$ [141]

$$+\left\{+\frac{2\,.\,233}{37}\left\{r_1-\lambda_1\right\}-\frac{51}{8}\gamma^2 s_{147}\right\}e_{\prime}^2\cos(4\,t-2\,z)$$ [142]

$\delta\,.\,r\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)$ and $\delta\int \mathrm{d}\,R$ may be obtained immediately from the preceding developments.

Developments required for the integration of the equation

$$\frac{\mathrm{d}\,\lambda'}{\mathrm{d}\,t}=h\,\frac{(1+s^2)}{r^2}-\frac{(1+s^2)}{r^2}\int\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda'}\right)\mathrm{d}\,t+\frac{(1+s^2)}{2\,r^2\,h}\left\{\int\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda'}\right)\mathrm{d}\,t\right\}^2$$

$$\mathrm{d}\,.\frac{\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda'}\right)}{\mathrm{d}\,s}=-\frac{2\,.\,2\,.\,3}{4}\,\frac{r^2}{r_{\prime}^3}\sin(2\,\lambda'-2\,\lambda_{\prime})\,s=-2\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda'}\right)s$$

$$=\frac{2\,m_{\prime}\,a^2}{a_{\prime}^3}\left\{-\frac{20}{27}\gamma\cos(2\,t-y)+\frac{20}{27}\gamma\cos(2\,t+y)+\frac{38}{17}\gamma\,e\cos(2\,t-x-y)\right.$$

[147] [148] [151]

$$-\frac{38}{17}\gamma\,e\cos(2\,t-x+y)-\frac{20}{27}\gamma\,e\cos(2\,t+x-y)+\frac{20}{27}\gamma\,e\cos(2\,t+x+y)$$

[152] [153] [154]

$$-\frac{70}{27}\gamma\,e_{\prime}\cos(2\,t-z-y)+\frac{70}{27}\gamma\,e_{\prime}\cos(2\,t-z+y)+\frac{10}{27}\gamma\,e_{\prime}\cos(2\,t+z-y)$$

[157] [158] [159]

$$-\frac{10}{27}\gamma e_{,} \cos(2t+z+y) - \frac{28}{15}\gamma e^2 \cos(2t-2x-y) + \frac{28}{15}\gamma^2 e^2 \cos(2t-2x+y)$$
[160] [163] [164]

$$-\frac{20}{27}\gamma e^2 \cos(2t+2x-y) + \frac{20}{27} e^2 \cos(2t+2x+y)$$
[165] [166]

$$+\frac{180}{23}\gamma e e_{,} \cos(2t-x-z-y) - \frac{180}{23}\gamma e e_{,} \cos(2t-x-z+y)$$
[169] [170]

$$+\frac{10}{27}\gamma e e_{,} \cos(2t+x+z-y) - \frac{10}{27}\gamma e e_{,} \cos(2t+x+z+y)$$
[171] [172]

$$-\frac{66}{59}\gamma e e_{,} \cos(2t-x+z-y) + \frac{66}{59}\gamma e e_{,} \cos(2t-x+z+y)$$
[175] [176]

$$-\frac{83}{32}\gamma e e_{,} \cos(2t+x-z-y) + \frac{83}{32}\gamma e e_{,} \cos(2t+x-z+y)$$
[177] [178]

$$-\frac{233}{37} e_{,}^2 \cos(2t-2z-y) + \frac{233}{37} e_{,}^2 \cos(2t-2z+y)$$
[181] [182]

$$\delta\left(\frac{dR}{d\lambda}\right) = \frac{2\,m_{,}\,a^2}{a_{,}^3}\left\{-\frac{40}{27} r_0' + \frac{38}{17} e^2 r_2' - \frac{20}{27} e^2 r_2' - \frac{70}{27} e_{,}^2 \left\{r_5' - \lambda_5\right\} + \frac{10}{27} e_{,}^2 \left\{r_5' + \lambda_5\right\}\right.$$

$$-\frac{28}{15} e^4 \left\{r_8 - \lambda_8\right\} - \frac{20}{27} e^4 \left\{r_8 + \lambda_8\right\} + \frac{180}{23} e^2 e_{,}^2 \left\{r_{11}' - \lambda_{11}\right\} + \frac{10}{27} e^2 e_{,}^2 \left\{r_{11}' + \lambda_{11}\right\}$$

$$\left. -\frac{66}{59} e^2 e_{,}^2 \left\{r_{14}' - \lambda_{14}\right\} - \frac{83}{32} e^2 e_{,}^2 \left\{r_{14}' + \lambda_{14}\right\}\right\} \sin 2t$$
[1]

$$+\left\{-\frac{20}{27}\left\{r_1' + \lambda_1\right\} - \frac{38}{17}\left\{r_1' + \lambda_1\right\} + \frac{28}{15} e^2 \left\{r_3' + \lambda_3\right\} - \frac{20}{27}\left\{r_3' + \lambda_3\right\}\right.$$

$$\left. + \frac{20}{27}\left\{r_4' + \lambda_4\right\} + \frac{19}{17}\gamma^2 s_{147} + \frac{10}{27}\gamma^2 s_{147}\right\} e \cos x$$
[2]

$$+\left\{+\frac{76}{17} r_0' - \frac{20}{27} r_2' + \frac{180}{23} e_{,}^2 \left\{r_5' - \lambda_5\right\} - \frac{66}{59} e_{,}^2 \left\{r_5' + \lambda_5\right\}\right\} e \sin(2t-x)$$
[3]

$$+\left\{-\frac{40}{27} r_0' - \frac{20}{27} r_2' - \frac{83}{32} e_{,}^2 \left\{r_5' - \lambda_5\right\} + \frac{10}{27} e_{,}^2 \left\{r_5' + \lambda_5\right\}\right\} e \sin(2t+x)$$
[4]

Development of $\delta\left(\frac{d R}{d \lambda'}\right)$.

$$+\left\{+\frac{10}{27}\left\{r_1'+\lambda_1\right\}+\frac{70}{27}\left\{r_1'+\lambda_1\right\}-\frac{66}{59}e^2\left\{r_3'+\lambda_3\right\}-\frac{180}{23}e^2\left\{r_3'+\lambda_3\right\}\right.$$
$$\left.+\frac{5}{27}\gamma^2 s_{147}-\frac{35}{27}\gamma^2 s_{147}\right\}e_i \sin z$$

[5]

$$+\left\{-\frac{140}{27}r_0'-\frac{233}{37}e_i^2\left\{r_5'-\lambda_5\right\}-\frac{20}{27}\left\{r_5'+\lambda_5\right\}\right\}e_i \sin(2t-z)$$

[6]

$$+\left\{+\frac{20}{27}r_0'-\frac{20}{27}\left\{r_5'-\lambda_5\right\}\right\}e_i \sin(2t+z)$$

[7]

$$+\left\{-\frac{20}{27}\left\{r_1'+\lambda_1\right\}+\frac{28}{15}\left\{r_1'+\lambda_1\right\}-\frac{7}{32}e^2\left\{r_3'+\lambda_3\right\}-\frac{20}{27}\left\{r_3'+\lambda_3\right\}\right.$$
$$-\frac{38}{17}\left\{r_4'+\lambda_4\right\}-\frac{20}{27}\left\{r_9'+\lambda_9\right\}+\frac{20}{27}\left\{r_{10}'+\lambda_{10}\right\}+\frac{10}{27}\gamma^2 s_{147}$$
$$\left.-\frac{14}{15}\gamma^2 s_{147}\right\}e^2 \sin 2x$$

[8]

$$+\left\{-\frac{56}{15}r_0'+\frac{38}{17}r_2'-\frac{105}{16}e_i^2\left\{r_5'-\lambda_5\right\}+\frac{15}{16}e_i^2\left\{r_5'+\lambda_5\right\}\right.$$
$$\left.-\frac{20}{27}\left\{r_8'+\lambda_8\right\}\right\}e^2 \sin(2t-2x)$$

[9]

$$+\left\{-\frac{40}{27}r_0'-\frac{21}{8}e_i^2\left\{r_5'-\lambda_5\right\}+\frac{3}{8}e_i^2\left\{r_5'+\lambda_5\right\}-\frac{20}{27}\left\{r_8'+\lambda_8\right\}\right\}e^2 \cos(2t+2x)$$

[10]

$$+\left\{+\frac{10}{27}\left\{r_1'+\lambda_1\right\}+\frac{180}{23}\left\{r_1'+\lambda_1\right\}+\frac{105}{16}e^2\left\{r_3'+\lambda_3\right\}+\frac{10}{27}\left\{r_3'+\lambda_3\right\}\right.$$
$$+\frac{70}{27}\left\{r_4'+\lambda_4\right\}-\frac{20}{27}\left\{r_6'+\lambda_6\right\}-\frac{38}{17}\left\{r_7'+\lambda_7\right\}-\frac{66}{59}e^2\left\{r_9'+\lambda_9\right\}$$
$$+\frac{83}{32}e^2\left\{r_{10}'+\lambda_{10}\right\}-\frac{20}{27}\left\{r_{12}'+\lambda_{12}\right\}-\frac{5}{27}\gamma^2 s_{147}$$
$$\left.+\frac{90}{23}\gamma^2 s_{147}\right\}e\,e_i \sin(x+z)$$

[11

$$+\left\{-\frac{360}{23}r_0'-\frac{70}{27}r_2'+\frac{153}{8}e_i^2\left\{r_5'-\lambda_5\right\}+\frac{38}{17}\left\{r_5'+\lambda_5\right\}\right.$$
$$\left.-\frac{20}{27}\left\{r_{11}'+\lambda_{11}\right\}\right\}e\,e_i \sin(2t-x-z)$$

[12]

Development of $\delta\left(\frac{d R}{d \lambda'}\right)$.

$$+\left\{+\frac{20}{27}r_0+\frac{10}{27}r_2'+\frac{3}{16}e^2\left\{r_3'-\lambda_3\right\}-\frac{20}{27}\left\{r_5'-\lambda_5\right\}\right.$$
$$\left.-\frac{20}{27}\left\{r_{11}'-\lambda_{11}\right\}\right\}e\,e_{\prime}\sin(2t+x+z)$$
[13]

$$+\left\{-\frac{83}{32}\left\{r_1'+\lambda_1\right\}+\frac{66}{59}\left\{r_1'+\lambda_1\right\}-\frac{15}{16}e^2\left\{r_3'+\lambda_3\right\}-\frac{70}{27}\left\{r_3'+\lambda_3\right\}\right.$$
$$-\frac{10}{27}\left\{r_4'+\lambda_4\right\}-\frac{38}{17}\left\{r_6'+\lambda_6\right\}-\frac{20}{27}\left\{r_7'-\lambda_7\right\}+\frac{180}{23}e^2\left\{r_9'-\lambda_9\right\}$$
$$-\frac{10}{27}e^2\left\{r_{10}'+\lambda_{10}\right\}+\frac{28}{15}e^2\left\{r_{12}'+\lambda_{12}\right\}-\frac{20}{27}\left\{r_{15}'+\lambda_{15}\right\}+\frac{20}{27}\left\{r_{16}'+\lambda_{16}\right\}$$
$$\left.+\frac{83}{64}\gamma^2 s_{147}-\frac{66}{59}\gamma^2 s_{147}\right\}e\,e_{\prime}\sin(x-z)$$
[14]

$$+\left\{-\frac{132}{59}r'_0+\frac{10}{27}r_2'+\frac{38}{17}\left\{r_5'-\lambda_5\right\}-\frac{20}{27}\left\{r_{14}'+\lambda_{14}\right\}\right\}e\,e_{\prime}\sin(2t-x+z)$$
[15]

$$+\left\{-\frac{83}{16}r'_0-\frac{51}{8}e_{\prime}^2\left\{r_5'-\lambda_5\right\}-\frac{20}{27}\left\{r_5'+\lambda_5\right\}-\frac{20}{27}\left\{r_{14}'-\lambda_{14}\right\}\right\}e\,e_{\prime}\sin(2t+x-z)$$
[16]

$$+\left\{+\frac{233}{37}\left\{r_1'+\lambda_1\right\}-\frac{153}{8}e^2\left\{r_3'+\lambda_3\right\}+\frac{10}{27}\left\{r_6'+\lambda_6\right\}+\frac{70}{27}\left\{r_7'+\lambda_7\right\}\right.$$
$$\left.-\frac{20}{27}\left\{r_{18}'+\lambda_{18}\right\}+\frac{20}{27}\left\{r_{19}'+\lambda_{19}\right\}-\frac{233}{74}\gamma^2 s_{147}\right\}e_{\prime}^2\sin 2z$$
[17]

$$+\left\{-\frac{466}{37}r'_0-\frac{845}{64}e_{\prime}^2\left\{r_5'-\lambda_5\right\}-\frac{70}{27}\left\{r_5'+\lambda_5\right\}-\frac{20}{27}\left\{r_{17}'-\lambda_{17}\right\}\right\}e_{\prime}^2\sin(2t-2z)$$
[18]

$$+\left\{+\frac{10}{27}\left\{r_5'+\lambda_5\right\}-\frac{e_{\prime}^2}{64}\left\{r_5'+\lambda_5\right\}-\frac{20}{27}\left\{r_{17}'-\lambda_{17}\right\}\right\}e_{\prime}^2\sin(2t+2z)$$
[19]

$$+\left\{+\frac{26}{69}\left\{r_1'+\lambda_1\right\}-\frac{3}{8}e^2\left\{r_3'+\lambda_3\right\}-\frac{10}{27}s_{147}\right\}\gamma^2\sin 2y$$
[62]

$$+\left\{-\frac{21}{16}e_{\prime}^2\left\{r_5'-\lambda_5\right\}+\frac{3}{16}e_{\prime}^2\left\{r_5'+\lambda_5\right\}\right\}\gamma^2\sin(2t-2y)$$
[63]

$$+\left\{+\frac{3}{8}\left\{r_1'+\lambda_1\right\}-\frac{26}{69}\left\{r_3'+\lambda_3\right\}-\frac{9}{8}s_{147}\right\}\gamma^2 e\sin(x-2y)$$
[65]

$$+\left\{-\frac{3}{8}\left\{r_1'+\lambda_1\right\}-\frac{3}{8}s_{147}\right\}\gamma^2 e\sin(x+2y)$$
[66]

Development of $\delta\left(\frac{dR}{d\lambda}\right)$.

$$+\left\{+\frac{3}{16}\left\{r_1'+\lambda_1\right\}+\frac{21}{16}s_{147}\right\}\gamma^2 e\sin(z-2y)$$
[71]

$$+\left\{+\frac{21}{16}\left\{r_1'+\lambda_1\right\}-\frac{3}{16}s_{147}\right\}\gamma^2 e_{\prime}\sin(z+2y)-\frac{26}{69}\left\{r_5'-\lambda_5\right\}\gamma^2 e_{\prime}\sin(2t-z-2y)$$
[72] [73]

$$-\frac{26}{69}\left\{r_5'+\lambda_5\right\}\gamma^2 e_{\prime}\sin(2t+z-2y)$$
[75]

$$+\left\{+\frac{3}{8}\left\{r_3'+\lambda_3\right\}+\frac{15}{16}s_{147}\right\}\gamma^2 e^2\sin(2x-2y)-\frac{3}{8}s_{147}\gamma^2 e^2\sin(2x+2y)$$
[77] [78]

$$+\left\{+\frac{3}{16}\left\{r_3'+\lambda_3\right\}-\frac{63}{16}s_{147}\right\}\gamma^2 e\,e_{\prime}\sin(x+z-2y)-\frac{3}{16}s_{147}\gamma^2 e\,e_{\prime}\sin(x+z+2y)$$
[83] [84]

$$+\frac{3}{8}\left\{r_5'+\lambda_5\right\}\gamma^2 e\,e_{\prime}\sin(2t-x-z-2y)$$
[85]

$$+\frac{3}{8}\left\{r_5'-\lambda_5\right\}\gamma^2 e\,e_{\prime}\sin(2t+x+z-2y)$$
[87]

$$+\left\{-\frac{21}{16}\left\{r_3'+\lambda_3\right\}+\frac{9}{16}s_{147}\right\}\gamma^2 e\,e_{\prime}\sin(x-z-2y)-\frac{21}{16}s_{147}\gamma^2 e\,e_{\prime}\sin(x-z+2y)$$
[89] [90]

$$+\frac{3}{8}\left\{r_5'-\lambda_5\right\}\gamma^2 e\,e_{\prime}\sin(2t-x+z-2y)$$
[91]

$$+\frac{3}{8}\left\{r_5'+\lambda_5\right\}\gamma^2 e\,e_{\prime}\sin(2t+x-z-2y)+\frac{51}{16}s_{147}\gamma^2 e_{\prime}^2\sin(2z-2y)$$
[93] [95]

$$+\frac{21}{16}\left\{r_5'+\lambda_5\right\}\gamma^2 e_{\prime}^2\sin(2t-2z-2y)+\frac{3}{16}\left\{r_5'-\lambda_5\right\}\gamma^2 e_{\prime}^2\sin(2t+2z-2y)$$
[97] [99]

$$+\frac{a}{a_{\prime}}\left\{-\frac{5.9}{8.4}\left\{r_1'+\lambda_1\right\}+\frac{3}{8}\left\{\frac{3}{4}r_1'+\frac{1}{4}\lambda_1\right\}+\frac{45.9}{16.4}e^2\left\{r_3'+\lambda_3\right\}\right.$$
$$-\frac{15}{16}e^2\left\{\frac{3}{4}r_3+\frac{1}{4}\lambda_3\right\}-\frac{9}{8}e_{\prime}^2\left\{\frac{3}{4}r_5'-\frac{1}{4}\lambda_5\right\}$$
$$\left.-\frac{3}{8}e_{\prime}^2\left\{\frac{3}{4}r_5'+\frac{1}{4}\lambda_5\right\}\right\}\sin t$$
[101]

$$+\frac{a}{a_{\prime}}\left\{+\frac{45.9}{16.4}\left\{r_1'+\lambda_1\right\}-\frac{3}{16}\left\{\frac{3}{4}r_1'+\frac{1}{4}\lambda_1\right\}+\frac{3}{8}\left\{\frac{3}{4}r_3'+\frac{1}{4}\lambda_3\right\}\right\}e\sin(t-x)$$
[102]

Development of $\delta\left(\frac{dR}{d\lambda}\right)$

$$+\frac{a}{a_{,}}\left\{-\frac{15.9}{16.4}\left\{r_1'+\lambda_1\right\}-\frac{15}{16}\left\{\frac{3}{4}r_1'+\frac{1}{4}\lambda_1\right\}-\frac{5.9}{8.4}\left\{r_3'+\lambda_3\right\}\right\}e\sin(t+x)$$
[103]

$$+\frac{a}{a_{,}}\left\{-\frac{25.9}{8.4}\left\{r_1'+\lambda_1\right\}+\frac{3}{8}\left\{\frac{3}{4}r_1'+\frac{1}{4}\lambda_1\right\}-\frac{3}{8}\left\{\frac{3}{4}r_5'+\frac{1}{4}\lambda_5\right\}\right\}e_{,}\sin(t-z)$$
[104]

$$+\frac{a}{a_{,}}\left\{+\frac{5.9}{8.4}\left\{r_1'+\lambda_1\right\}+\frac{9}{8}\left\{\frac{3}{4}r_1'+\frac{1}{4}\lambda_1\right\}-\frac{3}{8}\left\{\frac{3}{4}r_5'+\frac{1}{4}\lambda_5\right\}\right\}e_{,}\sin(t+z)$$
[105]

$$-\frac{a}{a_{,}}\frac{3}{16}\left\{\frac{3}{4}r_3'+\frac{1}{4}\lambda_3\right\}e^2\sin(t-2x)-\frac{a}{a_{,}}\frac{15.9}{16.4}\left\{r_3'+\lambda_3\right\}e^2\sin(t+2x)$$
[106] [107]

$$+\frac{a}{a_{,}}\left\{+\frac{3}{8}\left\{\frac{3}{4}r_3'+\frac{1}{4}\lambda_3\right\}+\frac{15}{16}\left\{\frac{3}{4}r_5'+\frac{1}{4}\lambda_5\right\}\right\}e\,e_{,}\sin(t-x-z)$$
[108]

$$+\frac{a}{a_{,}}\left\{+\frac{5.9}{8.4}\left\{r_3'+\lambda_3\right\}+\frac{3}{16}\left\{\frac{3}{4}r_5'-\frac{1}{4}\lambda_5\right\}\right\}e\,e_{,}\sin(t+x+z)$$
[109]

$$+\frac{a}{a_{,}}\left\{+\frac{9}{8}\left\{\frac{3}{4}r_3'+\frac{1}{4}\lambda_3\right\}+\frac{15}{16}\left\{\frac{3}{4}r_5'-\frac{1}{4}\lambda_5\right\}\right\}e\,e_{,}\sin(t-x+z)$$
[110]

$$+\frac{a}{a_{,}}\left\{-\frac{25.9}{8.4}\left\{r_3'+\lambda_3\right\}+\frac{3}{16}\left\{\frac{3}{4}r_5'+\frac{1}{4}\lambda_5\right\}\right\}e\,e_{,}\sin(t+x-z)$$
[111]

$$-\frac{a}{a_{,}}\frac{9}{8}\left\{\frac{3}{4}r_5'+\frac{1}{4}\lambda_5\right\}e_{,}^2\sin(t-2z)-\frac{a}{a_{,}}\frac{3}{8}\left\{\frac{3}{4}r_5'-\frac{1}{4}\lambda_5\right\}e_{,}^2\sin(t+2z)$$
[112] [113]

$$+\frac{a}{a_{,}}\left\{-\frac{3}{8}\left\{\frac{3}{4}r_1-\frac{1}{4}\lambda_1\right\}+\frac{3}{16}e^2\left\{\frac{3}{4}r_3'-\frac{1}{4}\lambda_3\right\}-\frac{25.9}{8.4}e_{,}^2\left\{r_5'-\lambda_5\right\}\right.$$
$$\left.+\frac{5.9}{8.4}e_{,}^2\left\{r_5'+\lambda_5\right\}\right\}\sin 3t$$
[116]

$$+\frac{a}{a_{,}}\left\{+\frac{15}{16}\left\{\frac{3}{4}r_1'-\frac{1}{4}\lambda_1\right\}-\frac{3}{8}\left\{\frac{3}{4}r_3'-\frac{1}{4}\lambda_3\right\}\right\}e\sin(3t-x)$$
[117]

$$+\frac{a}{a_{,}}\frac{3}{16}\left\{\frac{3}{4}r_1'-\frac{1}{4}\lambda_1\right\}e\sin(3t+x)$$
[118]

$$+\frac{a}{a_{,}}\left\{-\frac{9}{8}\left\{\frac{3}{4}r_1'-\frac{1}{4}\lambda_1\right\}-\frac{5.9}{8.4}\left\{r_5'+\lambda_5\right\}\right\}e_{,}\sin(3t-z)$$
[119]

Development of $\delta\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda'}\right)$.

$$+\frac{a}{a_{\prime}}\left\{-\frac{3}{8}\left\{\frac{3}{4}r_1{}'-\frac{1}{4}\lambda_1\right\}-\frac{5.9}{8.4}\left\{r_5{}'-\lambda_5\right\}\right\}e_{\prime}\sin(3t+z)$$ [120]

$$+\frac{a}{a_{\prime}}\frac{15}{16}\left\{\frac{3}{4}r_3{}'-\frac{1}{4}\lambda_3\right\}e^2\sin(3t-2x)$$ [121]

$$+\frac{a}{a_{\prime}}\left\{-\frac{9}{8}\left\{\frac{3}{4}r_3{}'-\frac{1}{4}\lambda_3\right\}+\frac{45.9}{16.4}\left\{r_5{}'+\lambda_5\right\}\right\}e\,e_{\prime}\sin(3t-x-z)$$ [123]

$$-\frac{a}{a_{\prime}}\frac{15.9}{16.4}\left\{r_5{}'-\lambda_5\right\}e\,e_{\prime}\sin(3t+x+z)$$ [124]

$$+\frac{a}{a_{\prime}}\left\{-\frac{3}{8}\left\{\frac{3}{4}r_3{}'-\frac{1}{4}\lambda_3\right\}+\frac{45.9}{16.4}\left\{r_5{}'-\lambda_5\right\}\right\}e\,e_{\prime}\sin(3t-x+z)$$ [125]

$$-\frac{a}{a_{\prime}}\frac{15.9}{16.4}\left\{r_5{}'+\lambda_5\right\}e\,e_{\prime}\sin(3t+x-z)$$ [126] $$-\frac{a}{a_{\prime}}\frac{25.9}{8.4}\left\{r_5{}'+\lambda_5\right\}e_{\prime}^2\sin(3t-2z)$$ [127]

$$+\frac{a}{a_{\prime}}\frac{5.9}{8.4}\left\{r_5{}'-\lambda_5\right\}e_{\prime}^2\sin(3t+2z)$$ [128]

$$+\left\{-\frac{20}{27}\left\{r_1{}'-\lambda_1\right\}-\frac{20}{27}e^2\left\{r_3{}'-\lambda_3\right\}+\frac{10}{27}\gamma^2 s_{147}\right\}\sin 4t$$ [131]

$$+\left\{+\frac{38}{17}\left\{r_1{}'-\lambda_1\right\}-\frac{20}{27}\left\{r_3{}'-\lambda_3\right\}-\frac{9}{8}\gamma^2 s_{147}\right\}e\sin(4t-x)$$ [132]

$$+\left\{-\frac{20}{27}\left\{r_1{}'-\lambda_1\right\}-\frac{20}{27}\left\{r_3{}'-\lambda_3\right\}-\frac{20}{27}\left\{r_4{}'-\lambda_4\right\}+\frac{3}{8}\gamma^2 s_{147}\right\}e\sin(4t+x)$$ [133]

$$+\left\{-\frac{70}{27}\left\{r_1{}'-\lambda_1\right\}-\frac{83}{32}e^2\left\{r_3{}'-\lambda_3\right\}+\frac{21}{16}\gamma^2 s_{147}\right\}e_{\prime}\sin(4t-z)$$ [134]

$$+\left\{+\frac{10}{27}\left\{r_1{}'-\lambda_1\right\}+\frac{10}{27}\left\{r_3{}'-\lambda_3\right\}+\frac{3}{16}\gamma^2 s_{147}\right\}e_{\prime}\sin(4t+z)$$ [135]

$$+\left\{-\frac{28}{15}\left\{r_1{}'-\lambda_1\right\}+\frac{38}{17}\left\{r_3{}'-\lambda_3\right\}+\frac{15}{16}\gamma^2 s_{147}\right\}e^2\sin(4t-2x)$$ [136]

$$+\left\{-\frac{20}{27}\left\{r_1{}'-\lambda_1\right\}-\frac{20}{27}\left\{r_4{}'-\lambda_4\right\}+\frac{3}{8}\gamma^2 s_{147}\right\}e^2\sin(4t+2x)$$ [137]

Development of $\delta\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right)$.

$$+\left\{+\frac{180}{23}\left\{r_1'-\lambda_1\right\}-\frac{70}{27}\left\{r_3'-\lambda_3\right\}-\frac{63}{16}\gamma^2 s_{147}\right\}e\,e_{\prime}\sin(4t-x-z)$$
[138]

$$+\left\{+\frac{10}{27}\left\{r_1'-\lambda_1\right\}+\frac{10}{27}\left\{r_4'-\lambda_4\right\}-\frac{3}{16}\gamma^2 s_{147}\right\}e\,e_{\prime}\sin(4t+x+z)$$
[139]

$$+\left\{-\frac{66}{59}\left\{r_1'-\lambda_1\right\}+\frac{10}{27}\left\{r_3'-\lambda_3\right\}+\frac{9}{16}\gamma^2 s_{147}\right\}e\,e_{\prime}\sin(4t-x-z)$$
[140]

$$+\left\{-\frac{83}{32}\left\{r_1'-\lambda_1\right\}+\frac{21}{16}\gamma^2 s_{147}\right\}e\,e_{\prime}\sin(4t+x-z)$$
[141]

$$+\left\{-\frac{233}{37}\left\{r_1'-\lambda_1\right\}+\frac{51}{16}\gamma^2 s_{147}\right\}e_{\prime}^2\sin(4t-2z)$$
[142]

In order to verify the developments which have been given, suppose

$$R=\frac{38\,m_{\prime}\,a^2}{17\,a_{\prime}^3}\,e\cos(2t-x)$$

$$\mathrm{r}\,\delta\,\frac{1}{r}=e_{\prime}\,r_5'\cos z \qquad \delta\lambda=e_{\prime}\,\lambda_5\sin z$$

neglecting δs,

$$\delta R=\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)\delta r+\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right)\delta\lambda=-a\left(\frac{\mathrm{d}R}{\mathrm{d}a}\right)\mathrm{r}\,\delta\,\frac{1}{r}+\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)\delta\lambda$$

t being used in the sense $n\,t-n_{\prime}\,t$.

$$\delta R=-\frac{2.38\,m_{\prime}\,a^2}{17\,a_{\prime}^3}\,e\,e_{\prime}\left\{\cos(2t-x)\,r_5'\cos z+\sin(2t-x)\,\lambda_5\sin z\right\}$$

$$=\frac{m_{\prime}\,a^2}{a_{\prime}^3}\,e\,e_{\prime}\left\{-\frac{38}{17}\,r_5'\cos(2t-x+z)-\frac{38}{17}\,r_5'\cos(2t-x-z)\right.$$

$$\left.+\frac{38}{17}\,\lambda_5\cos(2t-x+z)-\frac{38}{17}\,\lambda_5\cos(2t-x-z)\right\}$$

$$=\frac{m_{\prime}\,a^2}{a_{\prime}^3}\left\{-\frac{38}{17}\left\{r_5'-\lambda_5\right\}e\,e_{\prime}\cos(2t-x+z)-\frac{38}{17}\left\{r_5'+\lambda_5\right\}e\,e_{\prime}\cos(2t-x-z)\right\}$$
[15] [12]

which terms are in fact given in the development of δR, p. 11 and p. 10.

Again

$$\delta \, \mathrm{d} \, R = \frac{2 \, . \, 38 \, m \, m_{,} \, a^2 \, e \, e_{,}}{17 \, a_{,}^3} \left\{ \cos (2 \, t - x) \, r_5{}' \sin z - \sin (2 \, t - x) \, \lambda_5 \cos z \right\}$$

$$= \frac{m \, m_{,} \, a^2 \, e \, e_{,}}{a_{,}^3} \left\{ + \frac{38}{17} \, r_5{}' \sin (2 \, t - x + z) - \frac{38}{17} \, r_5{}' \sin (2 \, t - x - z) \right.$$

$$\left. - \frac{38}{17} \, \lambda_5 \sin (2 \, t - x + z) - \frac{38}{17} \, \lambda_5 \sin (2 \, t - x - z) \right\}$$

$$= \frac{m \, m_{,} \, a^2}{a_{,}^3} \left\{ + \frac{38}{17} \left\{ r_5{}' - \lambda_5 \right\} e \, e_{,} \sin (2 \, t - x + z) + \frac{38}{17} \left\{ r_5{}' + \lambda_5 \right\} e \, e_{,} \sin (2 \, t - x - z) \right\}$$

[15] [12]

which terms are given in the development of $\delta \, \mathrm{d} \, R$, p. 18.

Similarly

$$\delta \left(\frac{\mathrm{d} \, R}{\mathrm{d} \, \lambda} \right) = - \frac{2 \, . \, 2 \, . \, 38 \, m_{,} \, a^2}{17 \, a_{,}^3} \, e \, e_{,} \left\{ - \sin (2 \, t - x) \, r_5{}' \cos z + \cos (2 \, t - x) \, \lambda_5 \sin z \right\}$$

$$= - \frac{2 \, . \, 38 \, m_{,} \, a^2}{17 \, a_{,}^3} \, e \, e_{,} \left\{ - r_5{}' \sin (2 \, t - x + z) - r_5{}' \sin (2 \, t - x - x) \right.$$

$$\left. + \lambda_5 \sin (2 \, t - x + z) - \lambda_5 \sin (2 \, t - x - z) \right\}$$

$$= \frac{2 \, m_{,} \, a^2}{a_{,}^3} \left\{ + \frac{38}{17} \left\{ r_5{}' - \lambda_5 \right\} e \, e_{,} \sin (2 \, t - x + z) + \frac{38}{17} \left\{ r_5{}' + \lambda_5 \right\} e \, e_{,} \sin (2 \, t - x - z) \right\}$$

[15] [12]

these terms are given in the development of $\delta \left(\frac{\mathrm{d} \, R}{\mathrm{d} \, \lambda} \right)$, p. 25 and p. 24.

Suppose $\left(\frac{\mathrm{d} \, R}{\mathrm{d} \, s} \right) = \frac{20 \, m_{,} \, a^2}{27 \, a_{,}^3} \, \gamma \sin (2 \, t + y) \qquad \delta \, s = \gamma \, s_{147} \sin (2 \, t - y)$

$$\left(\frac{\mathrm{d} \, R}{\mathrm{d} \, s} \right) \delta \, s = \frac{20 \, m_{,} \, a^2}{27 \, a_{,}^3} \, \gamma^2 \, s_{147} \sin (2 \, t + y) \sin (2 \, t - y)$$

$$= \frac{m_{,} \, a^2}{a_{,}^3} \left\{ - \frac{10}{27} \, s_{147} \, \gamma^2 \cos 4 \, t + \frac{10}{27} \, s_{147} \, \gamma^2 \cos 2 \, y \right\}$$

[131] [62]

which terms are found in the development of $\delta \, R$.

$$\mathrm{d} \, . \left(\frac{\mathrm{d} \, R}{\mathrm{d} \, s} \right) \delta \, s = - \frac{2 \, . \, 20 \, m \, m_{,} \, a^2}{27 \, a_{,}^3} \, \gamma^2 \, s_{147} \sin (2 \, t + y) \cos (2 \, t - y)$$

$$= \frac{m \, m_{,} \, a^2}{a_{,}^3} \left\{ - \frac{20}{27} \, s_{147} \, \gamma^2 \sin 4 \, t + \frac{20}{27} \, s_{147} \, \gamma^2 \sin 2 \, y \right\}$$

[131] [62]

which terms are found in the development of $\delta\,\mathrm{d}\,R$. These terms are in fact multiplied by n which is equal to m if n be taken equal to unity.

$$\frac{\mathrm{d}\,.\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda}\right)}{\mathrm{d}\,s}\,\delta\,s = \frac{2\,.\,20\,m_{,}\,a^2}{27\,a_{,}^3}\,\gamma^2\,s_{147}\cos\left(2\,t + y\right)\sin\left(2\,t - y\right)$$

$$= \frac{2\,m\,a_{,}^2}{a_{,}^3}\left\{+\frac{10}{27}s_{147}\gamma^2\sin 4\,t - \frac{10}{27}\,s_{147}\,\gamma^2\sin 2\,y\right\}$$
$$[131] \qquad [62]$$

which terms are found in the development of $\delta\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda}\right)$.

$$\frac{1}{4}\left\{\int\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda}\right)\mathrm{d}\,t\right\}^2 = \frac{m_{,}^2\,a^4}{a_{,}^6}\left\{\frac{9}{128\,(1-m)^2} + \frac{81\,e^2}{32\,(2-2\,m-c)^2} + \frac{9\,e^2}{32\,(2-2\,m+c)^2}\right.$$

$$+ \frac{441\,e_{,}^2}{128\,(2-3\,m)^2} + \frac{9\,e^2}{128\,(2-m)^2} + \frac{9}{16\,(1-m)^2}\,\frac{a^2}{a_{,}^2} + \frac{9}{16\,(1-m)^2}\,\frac{a^2}{a_{,}^2}\cos 2\,t$$
$$[1]$$

$$-\left\{\frac{9}{4\,(2-2\,m-c)} - \frac{3}{4\,(2-2\,m+c)}\right\}\frac{3}{8\,(1-m)}\,e\cos x$$
$$[2]$$

$$-\left\{\frac{3}{8\,(2-m)} - \frac{21}{8\,(2-3\,m)}\right\}\frac{3}{8\,(1-m)}\,e_{,}\cos z$$
$$[5]$$

$$+\left\{-\frac{27}{16\,(2-2\,m-c)\,(2-2\,m+c)} + \frac{45}{64\,(1-m)\,(2-2\,m-2\,c)}\right.$$

$$\left.-\frac{9}{32\,(1-m)\,(2-2\,m+2\,c)}\right\}e^2\cos 2\,x$$
$$[8]$$

$$+\left\{-\left\{\frac{3}{8\,(2-m+c)} + \frac{63}{8\,(2-3\,m-c)}\right\}\frac{3}{8\,(1-m)} + \frac{63}{32\,(2-2\,m+c)\,(2-3\,m)}\right.$$

$$\left.+\frac{9}{32\,(2-2\,m-c)\,(2-m)}\right\}e\,e_{,}\cos\left(x+z\right)$$
$$[11]$$

$$+\left\{-\left\{\frac{21}{8\,(2-3\,m+c)} + \frac{9}{8\,(2-m-c)}\right\}\frac{3}{8\,(1-m)} - \frac{189}{32\,(2-3\,m)\,(2-2\,m-c)}\right.$$

$$\left.-\frac{9}{32\,(2-m)\,(2-2\,m+c)}\right\}e\,e_{,}\cos\left(x-z\right)$$
$$[14]$$

$$+\left\{-\frac{63}{64\,(2-m)\,(2-3\,m)} + \frac{153}{64\,(1-m)\,(2-4\,m)}\right\}e_{,}^2\cos 2\,z$$
$$[17]$$

$$+\left\{\frac{9}{128(1-m)^2}-\frac{27e^2}{16(2-2m+c)(2-2m-c)}-\frac{63e_{\prime}^2}{64(2-m)(2-3m)}\right\}\cos 4t$$
[131]

$$-\frac{27}{32(1-m)(2-2m-c)}e\cos(4t-x)+\frac{9}{32(1-m)(2-2m+c)}e\cos(4t+x)$$
[132] [133]

$$+\frac{63}{64(1-m)(2-3m)}e_{\prime}\cos(4t-z)-\frac{9}{64(1-m)(2-m)}e_{\prime}\cos(4t+z)$$
[134] [135]

$$+\left\{+\frac{45}{64(1-m)(2-2m-2c)}+\frac{81}{32(2-2m-c)^2}\right\}e^2\cos(4t-2x)$$
[136]

$$+\left\{+\frac{9}{32(1-m)(2-2m+2c)}+\frac{9}{32(2-2m+c)^2}\right\}e^2\cos(4t+2x)$$
[137]

$$+\left\{-\frac{189}{64(1-m)(2-3m-c)}-\frac{189}{32(2-2m+c)(2-3m)}\right\}ee_{\prime}\cos(4t-x-z)$$
[138]

$$+\left\{-\frac{9}{64(1-m)(2-m+c)}-\frac{9}{32(2-2m+c)(2-m)}\right\}ee_{\prime}\cos(4t+x+z)$$
[139]

$$+\left\{-\frac{27}{64(1-m)(2-m-c)}+\frac{27}{32(2-2m-c)(2-m)}\right\}ee_{\prime}\cos(4t-x+z)$$
[140]

$$+\left\{-\frac{63}{64(1-m)(2-m+c)}+\frac{63}{32(2-2m+c)(2-3m)}\right\}ee_{\prime}\cos(4t+x-z)$$
[141]

$$+\left\{+\frac{153}{64(1-m)(2-4m)}+\frac{441}{128(2-3m)^2}\right\}e_{\prime}^2\cos(4t-2z)$$
[142]

$$+\frac{9}{128(2-m)^2}e_{\prime}^2\cos(4t+2z)$$
[143]

$$\frac{h(1+s^2)}{r^2}=\frac{h}{\mathrm{r}^2}+\frac{2h}{\mathrm{r}}\delta.\frac{1}{r}+h\left(\delta\frac{1}{r}\right)^2+\frac{2hs}{\mathrm{r}^2}\delta s+\frac{4hs}{\mathrm{r}}\delta.\frac{1}{r}\delta s+\frac{h\,\delta s^2}{\mathrm{r}^2}$$

Developments which are required when the cube of the disturbing force is considered.

Neglecting in R the terms multiplied by $\frac{a^3}{a^4}$ and by s^2, and omitting the factor $m_{\prime}$,

$$R=-\frac{r^2}{4r_{\prime}^3}\left\{1+3\cos(2\lambda-2\lambda_{\prime})\right\}$$

$$R = -\frac{(r+\delta r)^2}{4 r_i^3}\left\{1 + 3\cos(2\lambda - 2\lambda_i + 2\delta\lambda)\right\}$$

$$= -\frac{\{1+3\cos(2\lambda-2\lambda_i)\}}{4 r_i^3}\left\{2 r \delta r + \delta r^2\right\} + \frac{3 r}{r_i^3}\sin(2\lambda - 2\lambda_i)\,\delta r\,\delta\lambda$$

$$+ \frac{3}{2}\frac{r^2}{r_i^3}(\cos(2\lambda - 2\lambda_i)\,(\delta\lambda)^2$$

$$\delta r = -r^2 \delta\frac{1}{r} + r^3\left(\delta\frac{1}{r}\right)^2$$

Neglecting the terms multiplied by $\delta\frac{1}{r}$ and $\delta\lambda$,

$$R = -3 r^2\frac{\{1+3\cos(2\lambda-2\lambda_i)\}}{4 r_i^3}\left(r\delta\frac{1}{r}\right)^2 - \frac{3 r^2}{r_i^3}\sin(2\lambda - 2\lambda_i)\left(r\delta\frac{1}{r}\right)\delta\lambda_i$$

$$+ \frac{3}{2}\frac{r^2}{r_i^3}\cos(2\lambda - 2\lambda_i)\,(\delta\lambda)^2$$

d R and $r\left(\frac{dR}{dr}\right)$ may be obtained from R as before.

$$\frac{dR}{d\lambda} = \frac{3}{2}\frac{r^2}{r_i^3}\sin(2\lambda - 2\lambda_i)$$

$$\frac{dR}{d\lambda} = \frac{3\{2 r\delta r + \delta r^2\}}{r_i^3}\sin(2\lambda - 2\lambda_i) + \frac{6\cos(2\lambda - 2\lambda_i)}{r_i^3}\,r\,\delta r\,\delta\lambda - \frac{3 r^2}{r_i^3}\sin(2\lambda - 2\lambda_i)\,(\delta\lambda)^2$$

Neglecting as before the terms multiplied by $\delta\frac{1}{r}$ and $\delta\lambda$,

$$= \frac{9}{2}\frac{r^2}{r_i^3}\sin(2\lambda - 2\lambda_i)\left(r\delta\frac{1}{r}\right)^2 - \frac{6 r^2\cos(2\lambda - 2\lambda_i)}{r_i^3}\left(r\delta\frac{1}{r}\right)\delta\lambda$$

$$- \frac{3 r^2}{r_i^3}\sin(2\lambda - 2\lambda_i)\,(\delta\lambda)^2$$

Retaining the terms depending on the cube of the disturbing force,

$$\frac{d^2 r^2}{2\,d t^2} - \frac{d^2 r^3\,\delta\frac{1}{2}}{d t^2} + \frac{3\,d^2 r^4\left(\delta.\frac{1}{r}\right)^2}{2\,d t^2} - \frac{2\,d^2 r^5\left(\delta\frac{1}{r}\right)^3}{d t^2} - \delta\frac{\mu}{\mu} + 2\int dR + r\left(\frac{dR}{dr}\right) = 0$$

$$\frac{d\lambda'}{dt} = \frac{h}{r'^2}\left\{1 - \frac{1}{h}\int\frac{dR}{d\lambda'}dt + \frac{1}{2h^2}\left\{\int\frac{dR}{d\lambda'}dt\right\}^2 - \frac{1.1}{2.4\,h^3}\left\{\int\frac{dR}{d\lambda'}dt\right\}^4\right.$$

$$\left. - \frac{1.1.3}{2.4.6\,h^5}\left\{\int\frac{dR}{d\lambda'}dt\right\}^6 + \&c.\right.$$

Fortunately this series does not appear to contain the quantity $\left\{\frac{dR}{d\lambda}dt\right\}^3$

The principal arguments in the expression for the longitude are those of which the indices and numerical coefficients in seconds (according to M. DAMOISEAU), ranged in their order of magnitude, are as follows:

$$\lambda' = \underset{[2]}{22639''{\cdot}70 \sin x} + \underset{[3]}{4589''{\cdot}61 \sin (2t - x)} + \underset{[1]}{2370''{\cdot}00 \sin 2t} + \underset{[8]}{768''{\cdot}72 \sin 2x} - \underset{[5]}{673''{\cdot}70 \sin z}$$

$$- \underset{[62]}{411''{\cdot}67 \sin 2y} + \underset{[9]}{211''{\cdot}57 \sin (2t - 2x)} + \underset{[12]}{207''{\cdot}09 \sin (2t - x - z)} + \underset{[4]}{192''{\cdot}22 \sin (2t + x)}$$

$$+ \underset{[6]}{165''{\cdot}56 \sin (2t - z)} + \underset{[14]}{147''{\cdot}74 \sin (x - z)} - \underset{[101]}{122''{\cdot}48 \sin t} - \underset{[11]}{109''{\cdot}27 \sin (x + z)}$$

The values of the quantities λ are, according to M. DAMOISEAU, p. 561,

*30.. $\lambda_1 = + {\cdot}0114901$
31.. $\lambda_3 = + {\cdot}405714$
32.. $\lambda_4 = + {\cdot}016992$
16.. $\lambda_5 = - {\cdot}194501$
33.. $\lambda_6 = + {\cdot}047798$
34.. $\lambda_7 = - {\cdot}0071657$
35.. $\lambda_9 = + {\cdot}341010$
36.. $\lambda_{10} = + {\cdot}023758$
19.. $\lambda_{11} = - {\cdot}57521$
41.. $\lambda_{12} = + 1{\cdot}090142$
42.. $\lambda_{13} = - {\cdot}015950$
18.. $\lambda_{14} = + {\cdot}77772$
39.. $\lambda_{15} = - {\cdot}15092$
40.. $\lambda_{16} = + {\cdot}077330$
17.. $\lambda_{17} = - {\cdot}12619$
43.. $\lambda_{18} = + {\cdot}13616$
44.. $\lambda_{19} = - {\cdot}0056734$
45.. $\lambda_{21} = + {\cdot}37647$
46.. $\lambda_{22} = + {\cdot}037323$
21.. $\lambda_{23} = - {\cdot}73617$
53.. $\lambda_{24} = + {\cdot}86288$
54.. $\lambda_{25} = - {\cdot}018236$
20.. $\lambda_{26} = + {\cdot}93487$
51.. $\lambda_{27} = + {\cdot}24475$
52.. $\lambda_{28} = + {\cdot}086383$
23.. $\lambda_{29} = - {\cdot}35736$
59.. $\lambda_{30} = + 2{\cdot}35733$
22.. $\lambda_{32} = + {\cdot}78995$
61.. $\lambda_{33} = - {\cdot}81190$

60.. $\lambda_{34} = + {\cdot}1630$
67.. $\lambda_{39} = + {\cdot}49834$
27.. $\lambda_{41} = - {\cdot}68253$
73.. $\lambda_{42} = + {\cdot}84004$
26.. $\lambda_{44} = + {\cdot}99754$
75.. $\lambda_{48} = + {\cdot}62872$
37.. $\lambda_{63} = + {\cdot}032768$
38.. $\lambda_{64} = - {\cdot}0034364$
49.. $\lambda_{67} = + {\cdot}0029421$
47.. $\lambda_{68} = - {\cdot}105155$
48.. $\lambda_{69} = - {\cdot}072464$
50.. $\lambda_{70} = - {\cdot}010897$
24.. $\lambda_{71} = - {\cdot}001424$
25.. $\lambda_{72} = + {\cdot}013168$
57.. $\lambda_{73} = + {\cdot}10392$
56.. $\lambda_{74} = - {\cdot}0060501$
55.. $\lambda_{75} = - {\cdot}047688$
58.. $\lambda_{76} = - {\cdot}0003559$
65.. $\lambda_{79} = - {\cdot}10332$
63.. $\lambda_{80} = - {\cdot}11921$
64.. $\lambda_{81} = - {\cdot}10332$
80.. $\lambda_{101} = - {\cdot}235981 \frac{a}{a_{,}}$
81.. $\lambda_{102} = - {\cdot}60389 \frac{a}{a_{,}}$
82.. $\lambda_{103} = - {\cdot}29509 \frac{a}{a_{,}}$
83.. $\lambda_{104} = - {\cdot}055072 \frac{a}{a_{,}}$

84.. $\lambda_{105} = + 2{\cdot}0147 \frac{a}{a_{,}}$
85.. $\lambda_{106} = - {\cdot}74945 \frac{a}{a_{,}}$
86 . $\lambda_{107} = - {\cdot}30746 \frac{a}{a_{,}}$
91.. $\lambda_{108} = - {\cdot}60668 \frac{a}{a_{,}}$
92.. $\lambda_{109} = + {\cdot}26150 \frac{a}{a_{,}}$
89.. $\lambda_{110} = + 4{\cdot}29 \frac{a}{a_{,}}$
100.. $\lambda_{116} = + {\cdot}00001927 \frac{a}{a_{,}}$
101.. $\lambda_{117} = - {\cdot}10679 \frac{a}{a_{,}}$
102.. $\lambda_{118} = + {\cdot}011593 \frac{a}{a_{,}}$
103.. $\lambda_{119} = + {\cdot}010326 \frac{a}{a_{,}}$
104.. $\lambda_{120} = + {\cdot}009179 \frac{a}{a_{,}}$
120.. $\lambda_{131} = + {\cdot}000071995$
121.. $\lambda_{132} = + {\cdot}0034139$
122.. $\lambda_{133} = - {\cdot}0000380$
123.. $\lambda_{134} = + {\cdot}0002367$
124.. $\lambda_{135} = - {\cdot}00002598$
125.. $\lambda_{136} = + {\cdot}050272$
126.. $\lambda_{137} = + {\cdot}0011766$
131.. $\lambda_{138} = + {\cdot}017477$
129.. $\lambda_{140} = - {\cdot}0025794$
127.. $\lambda_{144} = + {\cdot}00037053$

* Indices of M. DAMOISEAU.

According to the value of the parallax given by M. DAMOISEAU, p. 573,

$r_1 = \cdot 00834$, $r_3 = \cdot 18350$, $r_4 = \cdot 01625$, $r_5 = \cdot 00547$, $r_6 = \cdot 03342$, $r_7 = \cdot 004525$, &c. nearly.

From the preceding values it appears that several of the quantities λ which correspond to arguments in the longitude depending on the cubes and fourth powers of the eccentricities are of the same order as those which correspond to the arguments 1, 3, &c.: hence in order to carry the development of δR and $\delta \frac{dR}{d\lambda}$, &c. to the terms depending on the cubes of the eccentricities, λ_{21}, λ_{23}, λ_{24}, &c. cannot be neglected when extreme accuracy is sought; and if the method which I have employed should be adopted, it will be necessary to extend very considerably the Table II. so as to embrace these quantities.

The advantages of this method appear to me by no means confined to the condition of taking into account all sensible quantities; a few lines of calculation suffice to obtain approximate results.

Thus neglecting the squares of the eccentricities,

$$R = m_{\prime}\left\{-\frac{1}{r_{\prime}} - \frac{a^2}{4a_{\prime}^3} - \frac{3}{4}\frac{a^2}{a_{\prime}^3}\cos 2t + \frac{a^2}{2a_{\prime}^3}e\cos x + \frac{9}{4}\frac{a^2}{a_{\prime}^3}e\cos(2t-x) - \frac{3}{4}\frac{a^2}{a_{\prime}^3}e\cos(2t+x)\right.$$

$$\left. - \frac{3}{4}\frac{a^2}{a_{\prime}^3}e_{\prime}\cos z - \frac{21}{8}\frac{a^2}{a_{\prime}^3}e_{\prime}\cos(2t-z) + \frac{3}{8}\frac{a^2}{a_{\prime}^3}e_{\prime}\cos(2t+z)\right\}$$

$$-r_0 - \frac{m_{\prime}a^3}{2ma_{\prime}^3} = 0$$

$$4(1-m)^2 r_1 - r_1 - \frac{3}{2}\frac{m_{\prime}}{\mu}\frac{a^3}{a_{\prime}^3}\left\{\frac{1}{1-m} + 1\right\} = 0$$

$$c^2\left\{1 - 3r_0\right\} - 1 + \frac{2m_{\prime}}{\mu}\frac{a^3}{a_{\prime}^3} = 0$$

$$(1-2m)^2\left\{r_3 - \frac{3}{2}r_1\right\} - r_3 + \frac{9}{2}\frac{m_{\prime}}{\mu}\frac{a^3}{a_{\prime}^3}\left\{\frac{1}{1-2m} + 1\right\} = 0$$

$$(3-2m)^2\left\{r_4 - \frac{3}{2}r_1\right\} - r_4 - \frac{3}{2}\frac{m_{\prime}}{\mu}\frac{a^3}{a_{\prime}^3}\left\{\frac{3}{3-2m} + 1\right\} = 0$$

$$m^2 r_5 - r_5 - \frac{3}{2}\frac{m_{\prime}}{\mu}\frac{a^3}{a_{\prime}^3} = 0$$

$$(2-3m)^2 r_6 - r_6 - \frac{21}{4}\frac{m_{\prime}}{\mu}\frac{a^3}{a_{\prime}^3}\left\{\frac{2}{2-3m} + 1\right\} = 0$$

$$(2-m)^2 r_7 - r_7 + \frac{3}{4}\frac{m_{\prime}}{\mu}\frac{a^3}{a_{\prime}^3}\left\{\frac{2}{2-m} + 1\right\} = 0$$

$$-\left\{1-2m\right\}^2 z_{147}+\frac{3r_1}{2}+z_{147}=0$$

$$\lambda=\frac{h}{a^2}\left\{1+2r_0\right\}t+\frac{2e(1+r_0)}{c}\sin x$$

$$+\left\{2r_1+\frac{3m_{,}a^3}{4(1-m)\mu a_{,}^3}\right\}\frac{1}{2(1-m)}\sin 2t$$

$$+\left\{2r_3+r_1-\left\{\frac{9}{2(1-m)}-\frac{3}{2(2-m)}\right\}\frac{m_{,}a^3}{\mu a_{,}^3}\right\}\frac{e}{(1-2m)}\sin(2t-x)$$

$$+\left\{2r_4+r_1-\left\{-\frac{3}{2(3-m)}-\frac{3}{2(2-m)}\right\}\frac{m_{,}a^3}{\mu a_{,}^3}\right\}\frac{e}{(3-m)}\sin(2t+x)$$

$$+\frac{2r_5e_{,}}{m}\sin z$$

$$+\left\{2r_6+\frac{21m_{,}a^3}{4(2-3m)\mu a_{,}^3}\right\}\frac{e_{,}}{(2-3m)}\sin(2t-z)$$

$$+\left\{2r_7-\frac{3m_{,}a^3}{4(2-m)\mu a_{,}^3}\right\}\frac{e_{,}}{(2-m)}\sin(2t+z)$$

The values of the constants assumed by M. DAMOISEAU are

$e=\cdot0548442$ $\quad e_{,}=\cdot0167927$ $\quad \gamma=\cdot0900684$ $\quad m=\cdot0748013$

Mém. Théor. Lun. p. 502.

Taking $m=\frac{3}{40}=\cdot075$

$$\frac{4\,.\,37^2}{40^2}r_1-r_1-\frac{3\,.\,77\,m_{,}a^3}{2\,.\,37\,\mu a_{,}^3}=0 \qquad r_1=\frac{600\,.\,77\,m_{,}a^3}{17\,.\,57\,.\,37\,\mu a_{,}^3}$$

$$\frac{34^2}{40^2}\left\{r_3-\frac{3}{2}r_1\right\}-r_3+\frac{9\,.\,74\,m_{,}a^3}{2\,.\,34\,\mu a_{,}^3}=0$$

$$r_3=300\left\{\frac{17\,.\,77}{37^2\,.\,57}-\frac{2}{17}\right\}\frac{m_{,}a^3}{\mu a_{,}^3}=\frac{300\,.\,133813\,m_{,}a^3}{1326561\,\mu a_{,}^3}$$

$$r_5=-\frac{40^2\,.\,3\,m_{,}a^3}{37\,.\,43\,.\,2\,\mu a_{,}^3}=-\frac{2400\,m_{,}a^3}{1591\,\mu a_{,}^3}$$

$$z_{147}=\frac{3\,.\,40^2\,r_1}{2\,.\,4\,.\,37\,.\,3}$$

$$\lambda_1=\left\{2r_1+\frac{3\,.\,40\,m_{,}a^3}{4\,.\,37\,\mu a_{,}^3}\right\}\frac{20}{37}$$

$$\lambda_3=\left\{2r_3+r_1-\left\{\frac{9\,.\,20}{37}-\frac{3\,.\,20}{77}\right\}\frac{m_{,}a^3}{\mu a_{,}^3}\right\}\frac{20}{17}$$

$$= \left\{ 2r_3 + r_1 - \frac{11640\, m_{\prime} a^3}{2849\, \mu\, a_{\prime}^3} \right\} \frac{20}{17}$$

$$\lambda_5 = - \frac{40^3\, m_{\prime}\, a^3}{37\,.\,43\, \mu\, a_{\prime}^3}$$

If $\frac{m_{\prime} a^3}{\mu\, a_{\prime}^3} = \frac{1}{178{\cdot}725}$, as NEWTON finds, Principia, vol. iv. p. 2, Glasgow edit.,

$$r_1 = {\cdot}007208 \qquad r_3 = {\cdot}16928 \qquad r_5 = -{\cdot}008437 \qquad z_{147} = {\cdot}03896$$
$$\lambda_1 = {\cdot}010244 \qquad \lambda_3 = {\cdot}40409 \qquad \lambda_5 = -{\cdot}22501 \qquad s_{147} = z_{147} + r_1 = {\cdot}04617$$

These values being substituted in the developments of δR, $\delta\, \mathrm{d} R$ &c. given in this paper, more accurate values may be found from the differential equations by a new integration. It would be shorter, but perhaps not quite so satisfactory, to assume the values of λ_1, λ_3, &c. given by M. DAMOISEAU in these substitutions.

Converting the coefficients of the arguments of longitude into sexagesimal seconds;

$$\lambda = \underset{2370}{2113''} \sin 2t + \underset{4589{\cdot}61}{4571''{\cdot}3} \sin(2t - x) - \underset{673{\cdot}7}{779''{\cdot}3} \sin z$$

The numbers underneath are the values according to M. DAMOISEAU.

The coefficient of the variation thus obtained (2113″ or 35′ 13″) agrees within three seconds with that found by NEWTON, vol. iv. p. 19, which is 35′ 10″. The approximation is in fact of the same order as that of NEWTON. NEWTON does not appear to have succeeded in determining the evection, the most considerable of all the lunar inequalities after the equation of the centre. The value assigned by him to the annual equation is 11′ 51″ or 711″ (corresponding to $e_{\prime} = {\cdot}0169166$); he has not however given the method by which it was obtained.

The equation

$$c^2 \{1 - 3r_0\} - 1 + \frac{m'}{\mu} \left\{ -\frac{a^3}{2a_{\prime}^3} b_{3,0} + \frac{a^2}{a_{\prime}^2} b_{3,1} \right\} = 0$$

since $r_0 = \frac{m_{\prime}}{\mu} \left\{ \frac{a^3}{2a_{\prime}^3} b_{3,0} - \frac{a^2}{2a_{\prime}^3} b_{3,1} \right\}$ (See Phil. Trans. 1831. p. 50.)

gives $c = 1 + \frac{m_{\prime}}{\mu} \left\{ \frac{a^3}{a_{\prime}^3} b_{3,0} - \frac{5a^2}{4a_{\prime}^2} b_{3,1} \right\}$

If $\frac{h}{a^2} \left\{ 1 + 2r_0 \right\} = \mathrm{n}$ or $n \left\{ 1 + 2r_0 \right\} = \mathrm{n}$

$$c\,n = \mathrm{n} \left\{ 1 - \frac{m_{\prime}}{\mu} \left\{ \frac{a^3}{a_{\prime}^3} b_{3,0} - \frac{a^2}{a_{\prime}^2} b_{3\,1} - \frac{a^3}{a_{\prime}^3} b_{3,0} + \frac{5}{4} \frac{a^2}{a_{\prime}^2} b_{3,1} \right\} \right\}$$

$$c n = \mathrm{n}\left\{1 - \frac{m_{\prime} a^2}{4 \mu a_{\prime}^2} b_{3,1}\right\}$$

$$= \mathrm{n}\left\{1 - \frac{3 m_{\prime}}{4 \mu} \frac{a^3}{a_{\prime}^3}\right\} \text{ nearly.}$$

This coincides with the first term of the expression, Math. Tracts, p. 59.

If $\sqrt{\frac{\mu}{a^3}}\left\{1 - \frac{m_{\prime}}{\mu} \frac{a^3}{a_{\prime}^3}\right\} = \sqrt{\frac{\mu}{\mathrm{a}^3}}$

$$a = \mathrm{a}\left\{1 - \frac{2 m_{\prime}}{3 \mu} \frac{a^3}{a_{\prime}^3}\right\}$$

$$\frac{1 + r_0}{a} = \frac{1 + \frac{2 m_{\prime}}{3 \mu} \frac{a^3}{a_{\prime}^3} - \frac{m_{\prime} a^3}{2 a_{\prime}^3}}{\mathrm{a}} = \frac{1 + \frac{m_{\prime}}{6 \mu} \frac{a^3}{a_{\prime}^3}}{\mathrm{a}}$$

The equation for determining z gives

$$\frac{d^2 z}{d t^2} + \frac{\mu z}{r^3} + \left(\frac{d R}{d z}\right) = 0$$

If $s = \gamma \sin (g n t + \varepsilon - \nu)$

$$- g^2 + 1 + 3 r_0 + \frac{m_{\prime}}{2 \mu} \frac{a^3}{a_{\prime}^3} b_{3,0} = 0$$

$$r_0 = \frac{m_{\prime}}{\mu}\left\{\frac{a^3}{2 a_{\prime}^3} b_{3,0} - \frac{a^2}{2 a_{\prime}^2} b_{3,1}\right\}$$

$$g^2 = 1 + \frac{m_{\prime}}{\mu}\left\{\frac{3 a^3}{2 a_{\prime}^3} b_{3,0} - \frac{3 a^2}{2 a_{\prime}^2} b_{3,1} + \frac{a^3}{2 a_{\prime}^3} b_{3,0}\right\} = 0$$

$$g = 1 + \frac{m_{\prime}}{\mu}\left\{\frac{a^3}{a_{\prime}^3} b_{3,0} - \frac{3}{4} \frac{a^2}{a_{\prime}^2} b_{3,1}\right\}$$

$$n = \mathrm{n}\{1 - 2 r_0\}$$

$$g n = \mathrm{n}\left\{1 + \frac{m_{\prime}}{\mu}\left\{\frac{a^3}{a_{\prime}^3} b_{3,0} - \frac{3}{4} \frac{a^2}{a_{\prime}^2} b_{3,1}\right\}\right\}\left\{1 - \frac{m_{\prime}}{\mu}\left\{\frac{a^3}{a_{\prime}^3} b_{3,0} - \frac{a^2}{a_{\prime}^2} b_{3,1}\right\}\right\}$$

$$= \mathrm{n}\left\{1 + \frac{m_{\prime}}{4 \mu} \frac{a^2}{a_{\prime}^2} b_{3,1}\right\}$$

$$= \mathrm{n}\left\{1 + \frac{3}{4} \frac{m_{\prime}}{\mu} \frac{a^3}{a_{\prime}^3}\right\} \text{ nearly.}$$

This also coincides with the first term of the expression, Math. Tracts, p. 59; and it appears that when the square of the disturbing force is neglected, the *mean motion* of the perihelium of a planet is retrograde and equal to the *mean motion* of its node taken with a contrary sign.

The equations

$$d \nu + \frac{r'^2 \sin (\lambda' - \nu)}{h^2 \tan \iota}\left\{(1 + s^2)\left(\frac{d R}{d s}\right) - r' s \left(\frac{d R}{d r'}\right) - \left(\frac{d R}{d \lambda'}\right)\left(\frac{d s}{d \lambda'}\right)\right\} d \lambda' = 0$$

$$d\iota + \frac{r'^2 \cos \iota^2 \cos (\lambda' - \nu)}{h^2} \left\{ (1 + s^2) \right\} \left(\frac{dR}{ds}\right) - r's \left(\frac{dR}{dr'}\right) - \left(\frac{dR}{d\lambda'}\right) \left(\frac{ds}{d\lambda'}\right) \Big\} d\lambda' = 0$$

(see Phil. Trans. 1830, p. 334), serve to verify some of the theorems of NEWTON in the third volume of the Principia.

In fact

$$R = -\frac{m_, r'^2}{4 r_,^3} \left\{ 1 + 3 \cos (2\lambda - 2\lambda_,) - 2 s^2 \right\}$$

$$(1 + s^2) \left(\frac{dR}{ds}\right) = m_, s (1 + s^2) \frac{r'^2}{r_,^3}$$

$$r's \left(\frac{dR}{dr'}\right) = -\frac{m_, r'^2}{2 r_,^3} \left\{ 1 + 3 \cos (2\lambda' - 2\lambda_,) - 2 s^2 \right\} s$$

$$\left(\frac{dR}{d\lambda'}\right) = \frac{3 m_, r'^2}{2 r_,^3} \sin (2\lambda' - 2\lambda_,) \qquad \frac{ds}{d\lambda'} = \tan \iota \cos (\lambda' - \nu)$$

neglecting s^3,

$$d\nu + \frac{m_, r'^4 \sin (\lambda - \nu)}{h^2 r_,^3} \left\{ \sin (\lambda - \nu) + \frac{\sin (\lambda - \nu)}{2} \left\{ 1 + 3 \cos (2\lambda - 2\lambda_,) \right\} \right.$$

$$\left. - \frac{3}{2} \cos (\lambda - \nu) \sin (2\lambda - 2\lambda_,) \right\} d\lambda = 0$$

$$d\nu + \frac{m_, r^4 \sin (\lambda - \nu)}{h^2 r_,^3} \left\{ \sin (\lambda - \nu) + 3 \cos (\lambda - \lambda_,) \left\{ \cos (\lambda - \lambda_,) \sin (\lambda - \nu) \right. \right.$$

$$\left. \left. - \sin (\lambda - \lambda_,) \cos (\lambda - \nu) \right\} - \sin (\lambda - \nu) \right\} d\lambda = 0$$

$$d\nu = \frac{3 m_, r^4}{h^2 r_,^3} \sin (\lambda - \nu) \cos (\lambda - \lambda_,) \sin (\lambda_, - \nu)\, d\lambda'$$

$$= \frac{3 m_, a^3}{\mu a_,^3} \sin (\lambda - \nu) \cos (\lambda - \lambda_,) \sin (\lambda_, - \nu)\, d\lambda \text{ nearly}$$

$$= \frac{1}{59{\cdot}575} \sin (\lambda - \nu) \cos (\lambda - \lambda_,) \sin (\lambda_, - \nu)\, d\lambda$$

Which agrees with the result of NEWTON, Prop. Lib. 3. "Est igitur velocitas nodorum ut IT × PH × AZ, sive ut contentum sub sinubus trium angulorum TPI, PTN et STN. ..... Sunto enim PK, PH et AZ prædicti tres sinus. Nempe PK sinus distantiæ Lunæ a quadraturâ, PH sinus distantiæ Lunæ a nodo et AZ sinus distantiæ nodi a Sole, et erit velocitas nodi ut contentum PK × PH × AZ."

Similarly

$$d\iota = \frac{3 m_, r^4 \cos \iota}{h^2 r_,^3} \sin \iota \cos (\lambda - \nu) \cos (\lambda - \lambda_,) \sin (\lambda_, - \nu)\, d\lambda$$

"Erit angulus GPg (seu inclinationis horariæ variatio) ad angulum $33'' \, 16''' \, 3''''$ ut IT × AZ × TG × $\frac{\mathrm{P}p}{\mathrm{PG}}$ ad AT cub." Prop. XXXIV.

The stability of the system requires that the quantities c and g, which are determined by quadratic equations, should be rational. This is the case in the Theory of the Moon.

In the Planetary Theory, by well known theorems,

$$\mathrm{d}\varepsilon = (1 - \sqrt{1 - e^2})\,\mathrm{d}\varpi + \frac{2\,a^2 n}{\mu}\left(\frac{\mathrm{d}R}{\mathrm{d}a}\right)\mathrm{d}t$$

$$\mathrm{d}\varpi = -an\,\frac{\sqrt{1-e^2}}{\mu\,e}\left(\frac{\mathrm{d}R}{\mathrm{d}e}\right)\mathrm{d}t \qquad \mathrm{d}\nu = \frac{an}{\mu\sin\iota\sqrt{1-e^2}}\left(\frac{\mathrm{d}R}{\mathrm{d}\iota}\right)\mathrm{d}t$$

Neglecting the terms which are periodical,

$$\frac{\mathrm{d}\varepsilon - \mathrm{d}\varpi}{\mathrm{d}t} = \frac{m_{\prime}}{\mu}\left\{\frac{a^3}{a_{\prime}^3}\,b_{3,0} - \frac{5}{4}\,\frac{a^2}{a_{\prime}^2}\,b_{3,1}\right\} = k = -\,{}^{*}\frac{7}{4}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^3}$$

$$\frac{\mathrm{d}\varepsilon - \mathrm{d}\nu}{\mathrm{d}t} = \frac{m_{\prime}}{\mu}\left\{\frac{a^3}{a_{\prime}^3}\,b_{3,0} - \frac{3}{4}\,\frac{a^2}{a_{\prime}^2}\,b_{3,1}\right\} = -\frac{m_{\prime}}{4\,\mu}\,\frac{a^3}{a_{\prime}^3}$$

which evidently coincides with the result given p. 38.

Considering the parallactic inequality,

$$(1-m)^2 r_{101} - r_{101} - \frac{3\,m_{\prime}\,a^4}{8\,\mu\,a_{\prime}^4}\left\{\frac{2}{(1-m)} + 3\right\} = 0$$

$$\lambda_{101} = \left\{2\,r_{101} - \frac{3}{8\,(1-m)}\,\frac{m_{\prime}\,a^4}{\mu\,a_{\prime}^4}\right\}\frac{1}{(1-m)}$$

$$r_{101} = -\,\frac{191\,.\,200}{77\,.\,37}\,\frac{m_{\prime}\,a^4}{\mu\,a_{\prime}^4}$$

$$\lambda_{101} = \left\{2\,r_{101} + \frac{3\,.\,5}{37}\,\frac{m_{\prime}\,a^4}{\mu\,a_{\prime}^4}\right\}\frac{40}{37}$$

which equations give $r_{101} = -\cdot 07521\,\frac{a}{a_{\prime}}$; and if the parallactic inequality $= 122''\cdot 38$ according to Burg, and $a = \frac{1}{57'}$ or $\frac{1}{3420''}$, $a_{\prime} = \frac{1}{12''\cdot 67}$, that is, if the moon's horizontal parallax $= 57'$, the sun's parallax, according to the preceding equations, is $12''\cdot 7$; which however differs widely from the accurate value $8''\cdot 54$.

When the square of the disturbing force is neglected, the variable part of the angle $t + z$ may be considered the same as that of the angle x, and there-

$$* \; b_{3,0} = 2\left\{1 + \left(\frac{3}{2}\right)^2 \frac{a^2}{a_{\prime}^2} + \&\mathrm{c.}\right\} \qquad b_{3,1} = \frac{3\,a}{a_{\prime}} + \frac{3\,.\,3\,.\,5}{2\,.\,4}\,\frac{a^3}{a_{\prime}^3} + \&\mathrm{c.}$$

fore they may be included in the same inequality, either in the expression for the parallax or in that for the mean longitude.

In the elliptic theory

$$\frac{h^2}{\mu \cos \iota^2} = a\,(1 - e'^2)$$

$$e'^2 = e^2 \{1 - \sin^2 \iota \sin^2 (\nu - \varpi)\}$$ See Phil. Trans. 1831, p. 56.

These equations of condition are true, however far the approximation be carried; provided only, that the arbitrary quantities e and $\sin \iota$ be determined so as not to contain the mass of the sun implicitly.

The determination of the coefficients of the arguments $t + z$, $t - x + z$, and $2t - 2x + 2z$ will require particular attention in the numerical calculation. According to the analysis of M. Poisson (Journal de l'Ecole Polytechnique, vol. viii. and Mémoires de l'Académie des Sciences, vol. i.), the coefficient of the argument $t - x + z$ in the quantity $\int \mathrm{d}\, R$ equals zero. Conversely therefore this theorem may furnish an equation of condition between some of the coefficients. According to M. Damoiseau, the coefficient of this argument in the expression for the longitude is only $2''{\cdot}05$, and the argument $2t - 2x + 2z$ is insensible. The expressions which I gave, Phil. Trans. 1830, p. 334, are well adapted for finding in the theory of the moon, in which the square of the disturbing force is so sensible, by means of the variation of the elliptic constants, the coefficient of any inequality which arises from the introduction of a small divisor, these expressions being true, however far the approximation is carried.

It may be seen in the authors themselves, or in the excellent history of physical astronomy by M. Gautier *, that the methods of Clairaut, D'Alembert and Euler, do not resemble in any respect those which I have employed. Both Clairaut and D'Alembert, by means of the differential equation of the second order in which the true longitude is the independent variable, obtained the expression for the reciprocal of the radius vector in terms of cosines of the true longitude. They substituted this value in the differential equation which determines the time, and obtained by integration the value of the mean motion in terms of sines of the true longitude. By the reversion of series they then found the true longitude in terms of sines of the mean motion. The method

* Essai Historique sur le Problème des Trois Corps, p. 53.

of EULER is not so simple, but is remarkable as introducing the employment of three rectangular coordinates and the decomposition of forces in the direction of three rectangular axes.

Although D'ALEMBERT and CLAIRAUT made use of the same differential equations, disguised under a different notation *, yet they did not arrive at these in the same manner, nor did they employ the same method of integration.

LAPLACE has pushed the approximations to a much greater extent; but his method coincides in all respects with that of CLAIRAUT.

In the method of CLAIRAUT, when the square of the disturbing force and the squares of the eccentricity and inclination are neglected, the equations employed are

$$\left\{\frac{d^2\frac{1}{r}}{d\lambda^2}+\frac{1}{r}\right\}\left\{1-\frac{2}{h^2}\int r^2\left(\frac{dR}{d\lambda}\right)d\lambda\right\}-\frac{1}{a}$$

$$-\frac{r}{h^2}\left\{r\left(\frac{dR}{dr}\right)-\frac{1}{r}\left(\frac{dR}{d\lambda}\right)\frac{dr}{d\lambda}\right\}=0$$

$$\frac{dR}{d\lambda}=\frac{3m_{\prime}r^2}{2r_{\prime}^3}\sin(2\lambda-2\lambda_{\prime})$$

$$\frac{dR}{dr}=-\frac{m_{\prime}r}{2r_{\prime}^3}\left\{1+3\cos(2\lambda-2\lambda_{\prime})\right\}$$

$$h^2=\mu a \qquad r=\frac{a}{1+e\cos(c\lambda-\varpi)} \qquad \frac{dr}{d\lambda}=ce\sin(c\lambda-\varpi)$$

$$\frac{d^2\frac{1}{r}}{d\lambda^2}+\frac{1}{r}-\frac{1}{a}-\frac{3m_{\prime}}{\mu a^2}\int\frac{r^4}{r_{\prime}^3}\sin(2\lambda-2\lambda_{\prime})\,d\lambda$$

$$+\frac{m_{\prime}}{2\mu}\frac{r^3}{a r_{\prime}^3}\left\{1+3\cos(2\lambda-2\lambda_{\prime})\right\}$$

$$+\frac{3m_{\prime}er^2}{2\mu r_{\prime}^3}\sin(2\lambda-2\lambda_{\prime})\sin(\lambda-\varpi)=0$$

In order to integrate this equation, the value of $\lambda_{\prime}$ in terms of λ must be substituted, which substitution is an operation by no means simple, and therefore liable to occasion error.

* The neglect by mathematicians of care in the selection of algebraical symbols is much to be regretted. " La clarté des idées augmente à mesure que l'on perfectionne les signes qui servent à les exprimer."

$$\lambda_{,} = m\lambda - 2me\sin(c\lambda - \varpi) + 2e_{,}\sin(cm\lambda - \varpi_{,}) + \&c.$$

The equation

$$h\,\mathrm{d}t = \mathrm{d}\lambda\left\{1 + \frac{1}{h^2}\int\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right)r^2\,\mathrm{d}\lambda\right\}$$

gives t in terms of λ, and by the reversion of series λ may afterwards be obtained in terms of t. The equation for determining the inequalities of latitude is

$$\left\{\frac{\mathrm{d}^2 s}{\mathrm{d}\lambda^2} + s\right\}\left\{1 - \frac{2}{h^2}\int\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right)r^2\,\mathrm{d}\lambda\right\}$$

$$+ \frac{r^2}{h^2}\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) - \frac{r^2 s}{h^2}\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right) - \frac{r^2}{h^2}\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right)\left(\frac{\mathrm{d}\lambda}{\mathrm{d}s}\right) = 0$$

$$\frac{\mathrm{d}R}{\mathrm{d}s} = \frac{3m_{,}}{2}\frac{r^2}{r_{,}^3}\left\{1 + \cos(2\lambda - 2\lambda_{,})\right\} \qquad \frac{\mathrm{d}s}{\mathrm{d}\lambda} = g\gamma\cos(g\lambda - \nu)$$

I have given these equations, (which are to be found in various works *,) for the convenience of reference.

On the Planetary Theory.

In a former paper I have shown how the coefficients of the terms in the disturbing function multiplied by the cubes of the eccentricities in some particular examples may be reduced by means of some transformations applied to the coefficients of the same function multiplied by the squares of the eccentricities. The form of the disturbing function thus obtained is I think simpler than that of the Méc. Cél. in the terms multiplied by the cubes of the eccentricities, although the advantage obtained by these reductions is not so great as in the case of the terms multiplied by the squares of the eccentricities. I have now given the *general* form of the transformations required, in case any one should think it worth while to extend to the cubes of the eccentricities the general expression for the disturbing function given in the Philosophical Transactions for 1831, p. 295.

The coefficient of $e\,e_{,}\cos(2nt - 4n_{,}t + \varpi + \varpi_{,})$ or $e\,e_{,}\cos(3t - x + z)$

$$= \frac{3a}{4.4a_{,}^2}b_{3,2} + \frac{3a}{2.4a_{,}^2}b_{3,4} + \frac{3.3a^2}{2.4.2a_{,}^3}b_{5,1} + \frac{3}{2.4}\frac{(3a^2 - a_{,}^2)\,\dot{a}\,a_{,}}{a_{,}^5}b_{5,2} - \frac{3.7a^2}{2.4a_{,}^3}b_{5,2}$$

* See The Mechanism of the Heavens, by Mrs. SOMERVILLE, p. 427.

$$+\frac{3}{2.4}\frac{(3a^2-a_i^2)}{a_i^5}b_{5,4}+\frac{3.3a^2}{2.4.2a_i^3}b_{5,5}$$

Changing b_3 into $-\frac{3}{4}b_5$, and b_5 into $-\frac{5}{6}b_7$, we have

$$-\frac{3.3}{2.4.4}\frac{a}{a_i^2}b_{5,2}-\frac{3.3}{4.2.4}\frac{a}{a_i^2}b_{5,4}-\frac{5.3.3}{6.2.4.2}\frac{a^2}{a_i^3}b_{7,1}-\frac{5.3}{6.2.4}\frac{(3a_i^2-a^2)aa_i}{a_i^5}b_{7,2}$$

$$+\frac{5.3.7}{6.2.4}\frac{a^2}{a_i^3}b_{7,3}-\frac{5.3}{6.2.4}\frac{(3a^2-a_i^2)aa_i}{a_i^5}b_{7,4}-\frac{5.3.3}{6.2.4.2}\frac{a^2}{a_i^3}b_{7,5}$$

$$=-\frac{3.3}{2.4.4}\frac{a}{a_i^2}b_{5,2}-\frac{3.3a}{4.2.4a_i^2}b_{5,4}-\frac{5.3.3}{6.2.4.2}\frac{a^2}{a_i^3}b_{7,1}-\frac{5.3.3}{6.2.4}\frac{(a^2+a_i^2)aa_i}{a_i^2}b_{7,2}$$

$$+\frac{5.3}{6.2}\frac{a^2}{a_i^3}b_{7,2}+\frac{5.3.7}{6.2.4}\frac{a^2}{a_i^3}b_{7,3}+\frac{5.3}{6.2.4}\frac{(a^2+a_i^2)aa_i}{a_i^5}b_{7,4}$$

$$-\frac{5.3}{6.2}\frac{a}{a_i^4}b_{7,4}-\frac{5.3.3}{6.2.4.2}\frac{a^2}{a_i^3}b_{7,5}$$

$$=-\frac{3.3}{2.4.4}\frac{a}{a_i^2}b_{5,2}-\frac{3.3}{4.2.4}\frac{a}{a_i^2}b_{5,4}-\frac{5.3}{4.4}\frac{a}{a_i^2}\left\{\frac{(a^2+a_i^2)}{a_i^2}b_{7,2}-\frac{a}{a_i}b_{7,1}-\frac{a}{a_i}b_{7,3}\right\}$$

$$+\frac{5.3}{6.2}\frac{a^3}{a_i^4}b_{7,2}+\frac{5.3}{6.2.4}\frac{a}{a_i^2}\left\{\frac{(a^2+a_i^2)}{a_i^2}b_{7,4}-\frac{a}{a_i}b_{7,3}-\frac{a}{a_i}b_{7,5}\right\}$$

$$-\frac{5.3.3}{2.4.4}\frac{a^2}{a_i^3}\left\{b_{7,1}-b_{7,3}\right\}+\frac{5}{2.4.4}\frac{a^2}{a_i^3}\left\{b_{7,3}-b_{7,5}\right\}-\frac{5.3}{6.2}\frac{a^3}{a_i^4}b_{7,4}$$

$$=-\frac{3.3}{2.4.4}\frac{a}{a_i^2}b_{5,2}-\frac{3.3}{4.2.4}\frac{a}{a_i^2}b_{5,4}-\frac{5.3}{4.4}\frac{a}{a_i^2}b_{5,2}+\frac{3.3}{6}\frac{a}{a_i^2}b_{5,3}$$

$$+\frac{5.3}{6.2.4}\frac{a}{a_i^2}b_{5,4}-\frac{2.3.3}{4.4}\frac{a}{a_i^2}b_{5,2}+\frac{4}{4.4}\frac{a}{a_i^2}b_{5,4}$$

$$=-\frac{75}{32}\frac{a}{a_i^2}b_{5,2}+\frac{3}{2}\frac{a^2}{a_i^3}b_{5,3}+\frac{9}{32}\frac{a}{a_i^2}b_{5,4}$$

Operating in the same way on all the terms in R multiplied by the squares of the eccentricities, we obtain finally the quantity

$$+\Sigma\left\{\frac{\{a^2e^2+a_i^2e_i^2\}}{32}b_{5,i}-\frac{3}{16}\frac{a}{a_i^2}\sin^2\frac{\iota_i}{2}\left\{b_{5,i-1}+b_{5,i+1}\right\}\right.$$

$$\left.-\frac{3}{32}\frac{a}{a_i^2}(e^2+e_i^2)\left\{ib_{5,i-1}-ib_{5,i+1}\right\}\right\}\cos it$$

$$+\Sigma\left\{\frac{\{2i+7\}}{64}\frac{a}{a_i^2}b_{5,i-1}+\frac{\{8i+13\}}{32}\frac{a^2}{a_i^3}b_{5,i}-\frac{\{18i+15\}}{64}\frac{a}{a_i^2}b_{5,i+1}\right\}e^2\cos(it+2x)$$

$$+\Sigma\left\{-\frac{\{2i-3\}}{32}\frac{a}{a_{\prime}^{2}}b_{5,i-1}-\frac{i}{2}\frac{a^{2}}{a_{\prime}^{3}}b_{5,i}+\frac{\{18i-21\}}{32}\frac{a}{a_{\prime}^{2}}b_{5,i+1}\right\}e\,e_{\prime}\cos(it+x+z)$$

$$+\Sigma\left\{\frac{\{6i+7\}}{32}\frac{a}{a_{\prime}^{2}}b_{5,i-1}-\frac{\{6i+9\}}{32}\frac{a}{a_{\prime}^{2}}b_{5,i+1}\right\}e\,e_{\prime}\cos(it+x-z)$$

$$+\Sigma\left\{\frac{\{18i-15\}}{64}\frac{a}{a_{\prime}^{2}}b_{5,i-1}-\frac{\{8i-13\}}{32\,a_{\prime}}b_{5,i}-\frac{\{2i-7\}}{64}\frac{a}{a_{\prime}^{2}}b_{5,i+1}\right\}e_{\prime}^{2}\cos(it+2z)$$

$$-\Sigma\frac{3}{8}\frac{a}{a_{\prime}^{2}}b_{5,i-1}\sin^{2}\frac{\iota_{\prime}}{2}\cos(it+2y)$$

The terms in R multiplied by the cubes of the eccentricities are equal to the preceding quantity multiplied by

$$-\frac{2\,a^{2}}{a_{\prime}^{2}}e\cos x+\frac{3\,a}{a_{\prime}}e\cos(t-x)+\frac{3\,a}{a_{\prime}}e_{\prime}\cos(t+z)-\frac{a}{a_{\prime}}e\cos(t+x)$$

$$-\frac{a}{a_{\prime}}e_{\prime}\cos(t-z)-2\,e_{\prime}\cos z\,;$$

$$+\left\{-\frac{9}{8}\frac{\{a^{2}e^{2}+a_{\prime}^{2}e_{\prime}^{2}\}}{a_{\prime}^{2}}+\frac{3}{8}\frac{a^{2}}{a_{\prime}^{2}}e^{2}\cos 2x-\frac{3}{4}\frac{a}{a_{\prime}}\left\{e^{2}+e_{\prime}^{2}+2\sin^{2}\frac{\iota_{\prime}}{2}\right\}\cos t\right.$$

$$+\frac{9}{16}\frac{a}{a_{\prime}}e^{2}\cos(t+2x)-\frac{9}{16}\frac{a}{a_{\prime}}e_{\prime}^{2}\cos(t-2z)+\frac{3}{16}\frac{a}{a_{\prime}}e^{2}\cos(t-2x)$$

$$+\frac{3}{16}\frac{a}{a_{\prime}}e_{\prime}^{2}\cos(t+2x)+\frac{27}{8}\frac{a}{a_{\prime}}e\,e_{\prime}\cos(t-x+z)-\frac{9}{8}\frac{a}{a_{\prime}}e\,e_{\prime}\cos(t+x+z)$$

$$-\frac{9}{8}\frac{a}{a_{\prime}}e\,e_{\prime}\cos(t-x-z)+\frac{3}{8}\frac{a}{a_{\prime}}e\,e_{\prime}\cos(t+x-z)+\frac{3}{2}\frac{a}{a_{\prime}}\sin^{2}\frac{\iota_{\prime}}{2}\cos(t+2y)$$

$$\left.+\frac{3}{8}e_{\prime}^{2}\cos 2z\right\}$$

$$\left\{\Sigma\left\{-\frac{a}{4\,a_{\prime}^{2}}b_{5,i-1}-\frac{a^{2}}{2\,a_{\prime}^{3}}b_{5,i}+\frac{3\,a}{4\,a_{\prime}^{2}}b_{5,i+1}\right\}e\cos(it+x)\right.$$

$$\left.+\Sigma\left\{\frac{3}{4}\frac{a}{a_{\prime}^{2}}b_{5,i-1}-\frac{1}{2\,a_{\prime}}b_{5,i}-\frac{a}{4\,a_{\prime}^{2}}b_{5,i+1}\right\}e_{\prime}\cos(it+z)\right\}$$

$$+\frac{1}{2\,a_{\prime}^{3}}\left\{\frac{a^{2}e^{3}}{4}\cos x-\frac{a^{2}e^{3}}{4}\cos 3x+\frac{3}{2}a\,a_{\prime}\,e\,e_{\prime}^{2}\cos(t-x)-a\,a_{\prime}\left\{\frac{3}{4}e^{3}+\frac{e\,e_{\prime}^{2}}{2}\right\}\cos(t+x)\right.$$

$$+\frac{2\,a\,a_{\prime}}{3}e^{3}\cos(t+3x)+\frac{a\,a_{\prime}}{12}e^{3}\cos(t-3x)+\frac{3\,a\,a_{\prime}}{2}e_{\prime}e^{2}\cos(t+z)$$

$$-\frac{9}{8}a\,a_{\prime}e^{2}e_{\prime}\cos(t+2x+z)-\frac{3}{8}a\,a_{\prime}e^{2}e_{\prime}\cos(t-2x+z)$$

$$- a\,a_{,}\left\{\frac{3}{4}\,e_{,}^{3} + \frac{e^{2}e_{,}}{2}\right\}\cos(t-z) + \frac{3}{8}\,a\,a_{,}\,e^{2}e_{,}\cos(t+2x-z)$$

$$-\frac{9}{8}\,a\,a_{,}\,e\,e_{,}^{2}\cos(t-x-2z) + \frac{3}{8}\,a\,a_{,}\,e\,e_{,}^{2}\cos(t+x-2z) + \frac{2}{3}\,e_{,}^{3}\cos(t-3z)$$

$$-\frac{3}{8}\,e\,e_{,}^{2}\cos(t-x+2z) + \frac{e\,e_{,}^{2}}{8}\cos(t+x+2z) + \frac{e_{,}^{3}}{12}\cos(t+3z)$$

$$-3\,a\,a_{,}\,e\sin^{2}\frac{\iota_{,}}{2}\cos(t+x-2y) + a\,a_{,}\,e\sin^{2}\frac{\iota_{,}}{2}\cos(t-x-2y)$$

$$-3\,a\,a_{,}\,e_{,}\sin^{2}\frac{\iota_{,}}{2}\cos(t+z-2y) + a\,a_{,}\,e_{,}\sin^{2}\frac{\iota_{,}}{2}\cos(t-z-2y)$$

$$+\frac{a_{,}^{2}e_{,}^{3}}{4}\cos z - \frac{a_{,}^{2}e_{,}^{3}}{4}\cos 3z\Big\}$$

$$\Big\{\frac{b}{2}{}_{3,0} + b_{3,1}\cos t + b_{3,2}\cos 2t + \text{\&c.}$$

$+$ terms independent of b.

Multiplying out, the coefficient of each term may be put in terms of $b_{5,i-2}$, $b_{5,i-1}$, $b_{5,i}$, $b_{5,i+1}$ and $b_{5,i+2}$.

The quantities $b_{1,0}$, $b_{1,1}$, from which all the other quantities b_3, b_5, &c. depend, may be obtained at once from Table IX. in the Exercices de Calc. Intégral, by M. Legendre, vol. iii. See also vol. i. p. 171. of the same work.

$$\left(1+\frac{a}{a_{,}}\right)b_{1,0} = \frac{4}{\pi}\int\frac{d\varphi}{\Delta} \qquad \left(1+\frac{a}{a_{,}}\right)b_{1,1} = \frac{2}{\pi}\int -\frac{d\varphi\cos 2\varphi}{\Delta}$$

the integrals being taken from $\varphi = 0$ to $\varphi = \frac{1}{2}\pi$.

$$\Delta = \sqrt{(1 - c^{2}\sin^{2}\varphi)}$$

$$c^{2} = \frac{4\,a\,a_{,}}{(a+a_{,})^{2}} = \frac{4\,\alpha}{(1+\alpha)^{2}},\quad \alpha^{*} \text{ being} = \frac{a}{a_{,}} \text{ as in the notation of the Méc. Cél.}$$

$$b_{1,0} = \frac{4}{\pi(1+\alpha)}\,F^{1} \qquad b_{1,1} = \frac{2}{\pi(1+\alpha)}\left\{\frac{2}{c^{2}}(F^{1}-E^{1}) - F^{1}\right\}$$

In the theory of Jupiter disturbed by Saturn, $\alpha = \cdot 54531725$; and hence in this instance if $c = \sin\theta$, $\theta = 72^{\circ}\,53'\,17''$.

By interpolation, I find from Table IX. p. 424,

$$F\,(72^{\circ}\,53'\,18'') = 2\cdot 6460986$$

* ρ in the notation of Woodhouse's Astronomy, vol. iii. p. 287.

and $b_{1,0} = 2{\cdot}180214$, which differs but slightly from the value of $b_{1,0}$ given by LAPLACE, viz. 2·1802348.

The equation

$$\lambda = \frac{h}{r^2}\left\{1 - \frac{1}{h}\int \frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda}\,\mathrm{d}\,t\right\}$$

or
$$\delta\,\lambda = \frac{2\,h}{r}\,\delta\,\frac{1}{r} - \frac{h}{r^2}\int \frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda}\,\mathrm{d}\,t$$

appears to me to give numerical results more simply than that made use of by LAPLACE,

$$\delta\,\lambda = \frac{\dfrac{2\,r\,\delta\,r + \mathrm{d}\,r\,\delta\,r}{a^2 n\,\mathrm{d}\,t} + \dfrac{a\,n}{\mu}\left\{3\int\!\!\int \mathrm{d}\,t\,\mathrm{d}'\,R + 2\int r\left(\dfrac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)\mathrm{d}\,t\right\}}{\sqrt{1 - e^2}}$$

See Théor. Anal. vol. i. p. 491.

When, however, that part of the inequality only is wanted which has a small coefficient in the denominator, as in the great inequality of Jupiter, the latter equation seems preferable, which thus reduces itself to

$$\delta\,\lambda = \frac{3\,a\,n}{\mu\,\sqrt{1 - e^2}}\int\!\!\int \mathrm{d}\,t\,\mathrm{d}'R$$

The apparent difference between the value of the coefficient given by this equation and the former, (see Phil. Trans. 1831, p. 290,) arises, no doubt, from part of the expression given by the former containing *implicitly* the same small quantity in the numerator.

It appears from the last Number of the Bulletin des Sciences Mathématiques, that M. CAUCHY, in a Memoir read before the Academy of Turin, has given "definite integrals which represent the coefficient of any given cosine in the development of R," by which means the calculation of any given inequality depending on a high power of the eccentricity is much facilitated. A similar method is alluded to by M. POISSON, Mémoires de l'Institut, vol. i. p. 50.

The reader is requested to make the following corrections.

Phil. Trans. 1830, p. 331, line 11, *for* $r' \left(\frac{dR}{dr'}\right)$, *read* $r' s \left(\frac{dR}{dr'}\right)$.

p. 334, line 14, *for* $s \left(\frac{dR}{ds}\right) - \left(\frac{dR}{d\lambda'}\right) \frac{dr'}{d\lambda'}$,

read $r' s \left(\frac{dR}{dr'}\right) - \left(\frac{dR}{d\lambda'}\right) \frac{ds}{d\lambda'}$.

p. 334, line 15, *for* $s \left(\frac{dR}{ds}\right)$, *read* $r' s \left(\frac{dR}{dr'}\right)$.

Phil. Trans. 1831, p. 234.

Table I.

Line.	Column.	*for*	*read*
8	7	27	− 27
8	10	......	40
17	4	34	− 34
18	62	92	98
35	4	58	55
65	3	77	− 80
146	9	163	−163
55	4	......	35
58	4	35	......
77	3	65	......
80	3	......	− 65
139	3	......	25
141	3	......	28
143	3	......	31
144	1	......	63
145	1	......	64
163	9	146	−146
164	8	147	......
165	8	......	147

Table II.

Line.	Column.	*for*	*read*
27	8	7	− 7
34	4	17	− 17
40	10	......	8

Addition to Table I. p. 277.

6	149	169	177

P. 271, *for* $s = \left\{z_{146} + \frac{e^2}{2} z_{150} + \frac{e^2}{2} z_{149}\right\} \gamma \sin y + \&c.$,

read $$s = \left\{z_{146} + \frac{e^2}{2} z_{150} + \frac{e^2}{2} {}_{149}\right\} \gamma \sin y$$

$$+ \left\{z_{147} + \frac{e^2}{2} z_{151} + \frac{e^2}{2} z_{153} - \frac{r_1}{2}\right\} \gamma \sin (2t - y)$$

$$+ \left\{z_{148} + \frac{e^2}{2} z_{152} + \frac{e^2}{2} z_{154} + \frac{r_1}{2}\right\} \gamma \sin (2t + y) + \&c.$$

P. 273, line 7, *for* $-\frac{3r_1}{2}$, *read* $+\frac{3r_1}{2}$.

P. 8 (of the preceding paper), line 10, *for* $\frac{20}{27}ee_{\prime}\cos(x+z)$, *read* $\frac{20}{27}ee_{\prime}\cos(x-z)$.

P. 10, line 9, *for* $\frac{105}{16}\left\{r_3'+\lambda_3\right\}$, *read* $\frac{105}{16}e^2\left\{r_3'+\lambda_3\right\}$.

P. 12, line 2, *for* $\frac{9}{8}r_3'$, *read* $\frac{9}{8}e^2r_3'$.

——— 3, *for* $\frac{3}{8}r_3'$, *read* $\frac{3}{8}e^2r_3'$.

———13, *for* $\frac{26}{69}\left\{r_5'-\lambda_5\right\}$, *read* $\frac{26}{69}\left\{r_5'+\lambda_5\right\}$.

P. 13, at foot, *insert*

$$\underset{[97]}{\frac{21}{16}\left\{r_5'+\lambda_5\right\}\gamma^2e_{\prime}^2\sin(2t-2z-2y)}-\underset{[99]}{\frac{3}{16}\left\{r_5'-\lambda_5\right\}\gamma^2e_{\prime}^2\sin(2t+2z-2y)}.$$

P. 14, line 13, *for* $\frac{3}{16}\left\{\frac{3}{2}r_3'-\frac{1}{2}\lambda_3\right\}+\frac{25.3}{8.2}\left\{r_5'-\lambda_5\right\}-\frac{5.3}{8.2}\left\{r_5'+\lambda_5\right\}$,

read $\frac{3}{16}e^2\left\{\frac{3}{2}r_3'-\frac{1}{2}\lambda_3\right\}+\frac{25.3}{8.2}e_{\prime}^2\left\{r_5'-\lambda_5\right\}-\frac{5.3}{8.2}e_{\prime}^2\left\{r_5'+\lambda_5\right\}$.

P. 15, line 3, *for* $\frac{5.3}{8.2}\left\{r_1'+\lambda_1\right\}$, *read* $\frac{5.3}{8.2}\left\{r_5'+\lambda_5\right\}$.

——— 4, *for* $\frac{5.3}{8.2}\left\{r_3'-\lambda_3\right\}$, *read* $\frac{5.3}{8.2}\left\{r_5'-\lambda_5\right\}$.

———13, *for* $\frac{20}{27}e^2\left\{r_3'-\lambda_3\right\}$, *read* $\frac{20}{27}\left\{r_3'-\lambda_3\right\}$.

P. 37, line 5, *for* $s_{147}=z_{147}+r_{\prime}=\cdot04617$, *read* $s_{147}=z_{147}-\frac{r_1}{2}=\cdot03536$.

P. 38, line 20, *for* retrograde, *read* direct.